Logical Atomicity in Iris: the Good, the Bad, and the Ugly

Ralf Jung

MPI-SWS, Germany

Iris Workshop, October 2019

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What is the right specification?

$$\mathtt{rec}\ \mathtt{inc}(x) = \mathtt{let}\ v = !x;$$
$$\mathtt{if}\ \mathtt{CAS}(x, v, v+1)\ \mathtt{then}\ v$$
$$\mathtt{else}\ \mathtt{inc}(x)$$

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Compare-and-swap

What is the right specification?

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$$\precsim$$

$$\lambda x.\, \texttt{FAA}(x, 1)$$

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$$\texttt{rec inc}(x) = \texttt{let}\ v = {!x};\ \texttt{if}\ \texttt{CAS}(x, v, v+1)\ \texttt{then}\ v\ \texttt{else inc}(x)$$

$$\precsim$$

Fetch-and-add

$$\lambda x.\ \texttt{FAA}(x, 1)$$

What is the right specification?

$$
\begin{aligned}
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\texttt{if}\ \texttt{CAS}(x, v, v+1)\ \texttt{then}\ v\\
\texttt{else}\ \texttt{inc}(x)
\end{aligned}
$$

$$\precsim$$

$$\lambda x.\ \texttt{FAA}(x, 1)$$

What is the right specification?

$\mathtt{rec}\ \mathtt{inc}(x) = \mathtt{let}\ v = !x;$

Common approach:

- Use contextual refinement as spec
- Use linearizability to prove it

$\sim$

$\lambda x.\ \mathtt{FAA}(x, 1)$

$$\frac{\dfrac{\texttt{inc} \precsim \lambda x.\, \texttt{FAA}(x, 1)}{???}}{\{P\}\ \texttt{client}[\texttt{inc}]\ \{Q\}}$$

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$$\frac{???}{\{P\}\ \texttt{client}[\lambda x.\ \texttt{FAA}(x, 1)]\ \{Q\}}$$

Specification for `FAA`:

$$\{\ell \mapsto v\}\ \texttt{FAA}(\ell, 1)\ \{\ell \mapsto v + 1\}$$

Specification for FAA:

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However, we also have:

$$\{\ell \mapsto v\}\ \texttt{incS}(\ell)\ \{\ell \mapsto v + 1\}$$

where $\texttt{incS} \triangleq \lambda x.\ \texttt{let}\ v = !x;\ x \leftarrow v + 1$

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but $\texttt{incS} \not\precsim \lambda x.\ \texttt{FAA}(x, 1)$!

Specification for `FAA`:

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There is something `FAA` has that `incS` does not:

but `incS` $\not\approx \lambda x.$ `FAA`$(x, 1)$:

Specification for FAA:

$$\{\ell \mapsto v\}\ \texttt{FAA}(\ell, 1)\ \{\ell \mapsto v + 1\}$$

There is something FAA has that incS does not: the invariant rule!

$$\frac{\{P * I\}\ \texttt{FAA}(x, 1)\ \{Q * I\}}{\boxed{I} \vdash \{P\}\ \texttt{FAA}(x, 1)\ \{Q\}}$$

but incS $\not\approx \lambda x.\, \texttt{FAA}(x, 1)$.

Key idea for logical atomicity

An operation is atomic if we can open invariants around it.

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An operation is atomic if we can open invariants around it.

How can we open invariants around `inc`(x)?

Logically atomic Hoare triples

1. Define $\langle x.\, P \rangle\; e\; \langle Q \rangle$

Logically atomic Hoare triples

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2. Prove $\langle v.\, \ell \mapsto v\rangle\, \texttt{inc}(\ell)\, \langle \ell \mapsto v + 1\rangle$

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4. Profit!

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1. Define $\langle x.\ P \rangle\ e\ \langle Q \rangle$
2. Prove $\langle v.\ \ell \mapsto v \rangle\ \texttt{inc}(\ell)\ \langle \ell \mapsto v + 1 \rangle$
3.

Plan for this talk:

Logical atomicity v0.1, v0.2, v1

4. Profit!

Logical Atomicity, v0.1: the basics

Weaker Specification

$$\begin{aligned}\texttt{rec}\ \texttt{inc}(x) = \texttt{let}\ v = !x;\\ \texttt{if}\ \texttt{CAS}(x, v, v+1)\ \texttt{then}\ v\\ \texttt{else}\ \texttt{inc}(x)\end{aligned}$$

Weaker Specification

$$\langle v.\, \ell \mapsto v \rangle \; \texttt{inc}(\ell) \; \langle \ell \mapsto v + 1 \rangle$$

$$\ell \mapsto v \Rrightarrow \exists \gamma.\; \Box\, \mathsf{IsCtr}(\ell, \gamma) * \mathsf{CtrV}(\gamma, v)$$

$$\mathsf{IsCtr}(\ell, \gamma) \vdash \langle v.\, \mathsf{CtrV}(\gamma, v) \rangle \; \texttt{inc}(\ell) \; \langle \mathsf{CtrV}(\gamma, v + 1) \rangle$$

Weaker Specification

Abstract predicate seals off direct access to ℓ

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$$\forall v.\ \{\ell \mapsto v\}\ \texttt{incS}(\ell)\ \{\ell \mapsto v + 1\}$$

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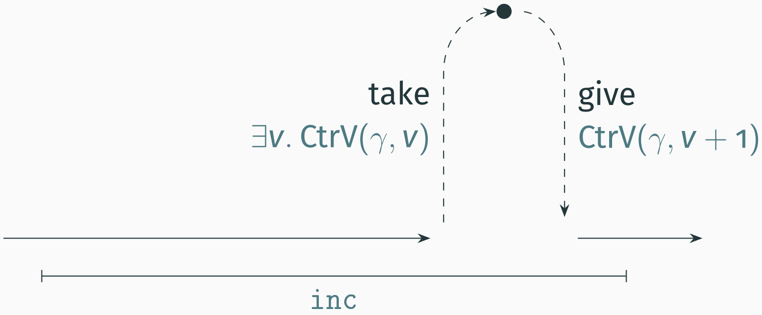

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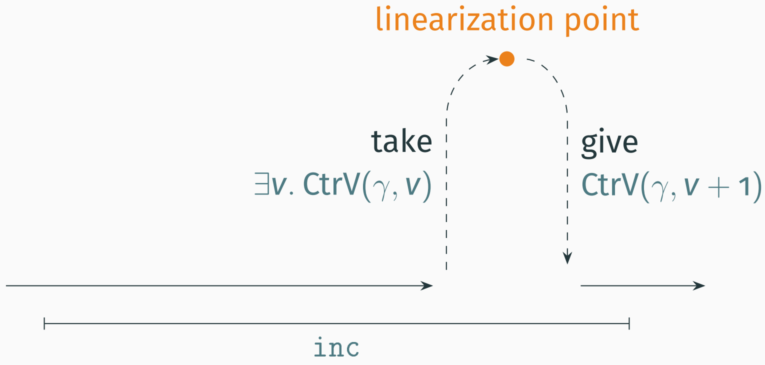
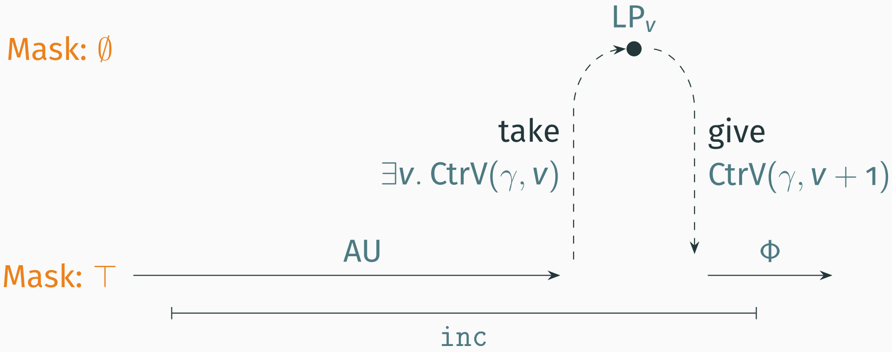


$$\langle v.\, \mathsf{CtrV}(\gamma, v)\rangle\ \texttt{inc}(\ell)\ \langle \mathsf{CtrV}(\gamma, v+1)\rangle^{\top} \triangleq$$

$$\forall \Phi.\, \mathsf{AU} \mathrel{-\!\!*} \mathsf{wp}\ \texttt{inc}(\ell)\ \{\Phi\}$$



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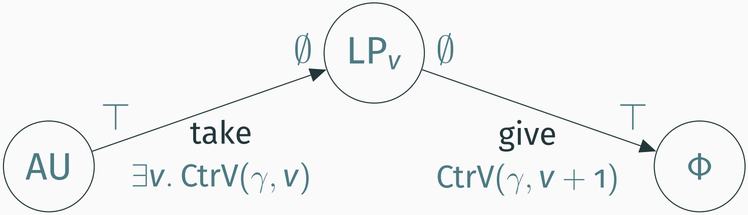
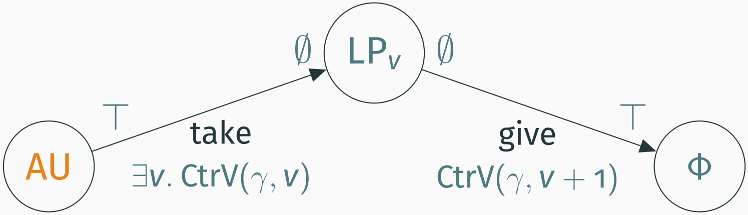


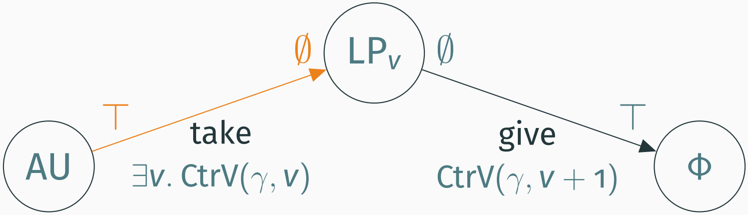
$$\forall \Phi.\ \mathsf{AU} \twoheadrightarrow* \mathsf{wp}\ \mathtt{inc}(\ell)\ \{\Phi\}$$

$\mathsf{AU} \triangleq$



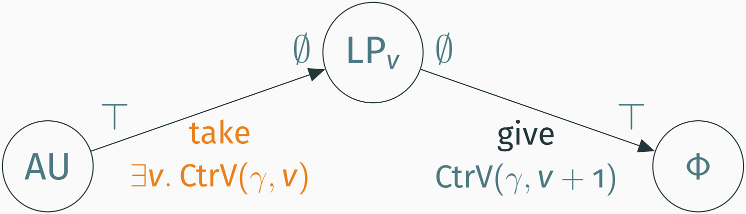
$$\forall \Phi.\, \mathsf{AU} \twoheadrightarrow \mathsf{wp}\; \mathtt{inc}(\ell)\; \{\Phi\}$$

$$\mathsf{AU} \triangleq {}^{\top}\!\Rrightarrow^{\emptyset}$$



$$\forall \Phi.\, \mathsf{AU} \mathrel{-\!\!*} \mathsf{wp}\ \texttt{inc}(\ell)\ \{\Phi\}$$

$$\mathsf{AU} \triangleq {}^{\top}\!\!\Rrightarrow^{\emptyset} \exists v.\, \mathsf{CtrV}(\gamma, v)$$



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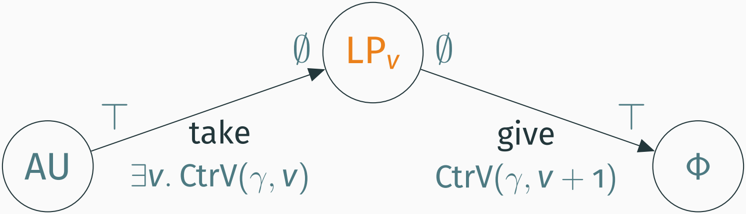
$$\mathsf{AU} \triangleq {}^{\top}\!\Rrightarrow^{\emptyset} \exists v.\ \mathsf{CtrV}(\gamma, v) * \mathsf{LP}_v$$



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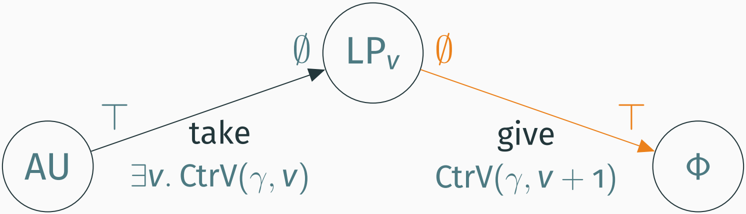

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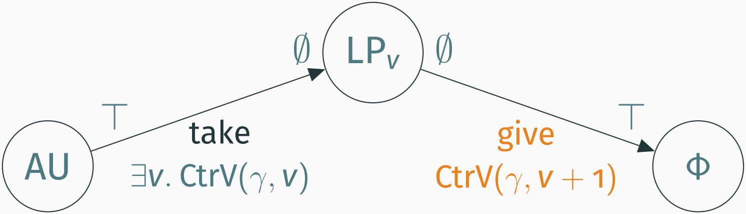

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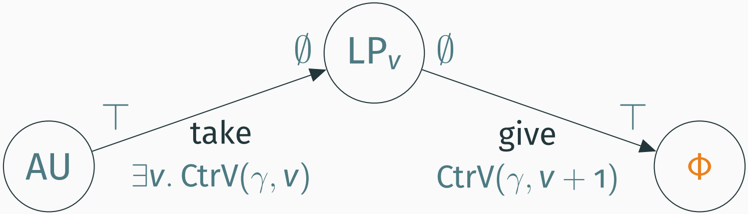

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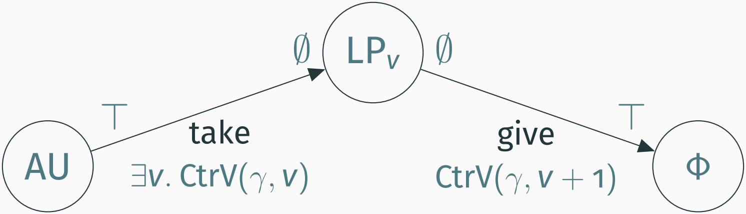

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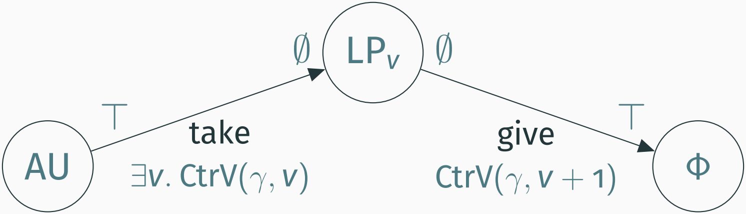
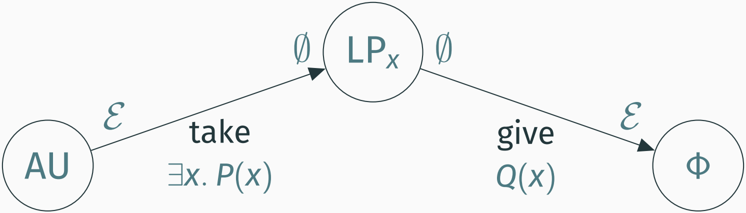
$$\mathsf{AU} \triangleq {}^{\top}\!\!\Rrightarrow^{\emptyset} \exists v.\ \mathsf{CtrV}(\gamma, v) * \left(\mathsf{CtrV}(\gamma, v+1)\ {}^{\emptyset}\!\!\equiv\!\!\!*^{\top}\ \Phi\right)$$



$$\langle x.\, P(x) \rangle\ e\ \langle Q(x) \rangle^{\mathcal{E}} \triangleq \forall \Phi.\ \mathsf{AU} \mathrel{-\!\!*} \mathsf{wp}\ e\ \{\Phi\}$$

$$\mathsf{AU} \triangleq {}^{\mathcal{E}}{\Rrightarrow}^{\emptyset}\, \exists x.\, P(x) * \left(Q(x)\ {}^{\emptyset}{\Rrightarrow\!\!*}^{\mathcal{E}}\ \Phi\right)$$



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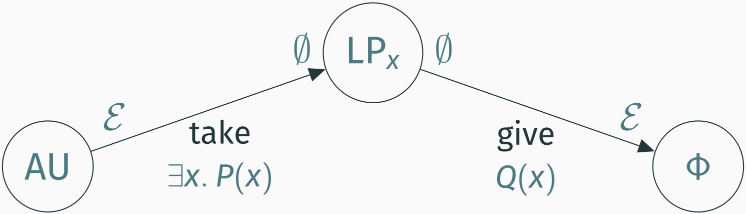

$$\langle x.\, P(x)\rangle\ e\ \langle Q(x)\rangle^{\mathcal{E}} \triangleq$$

$$\forall \Phi.\ \left({}^{\mathcal{E}}{\Rrightarrow}^{\emptyset}\, \exists x.\, P(x) * \left(Q(x) \mathrel{-\!\!*} {}^{\emptyset}{\Rrightarrow}^{\mathcal{E}}\, \Phi\right)\right) \mathrel{-\!\!*} \mathsf{wp}_{\top}\ e\ \{\Phi\}$$

$$\forall x.\ \{P(x)\}\ e\ \{Q(x)\}_{\top} \iff$$

$$\forall \Phi.\ \left(\exists x.\, P(x) * \left(Q(x) \mathrel{-\!\!*} \Phi\right)\right) \mathrel{-\!\!*} \mathsf{wp}_{\top}\ e\ \{\Phi\}$$

$$\langle x.\, P(x)\rangle\ e\ \langle Q(x)\rangle^{\mathcal{E}} \triangleq \forall \Phi.\ \mathsf{AU} \mathrel{-\!\!*} \mathsf{wp}\ e\ \{\Phi\}$$

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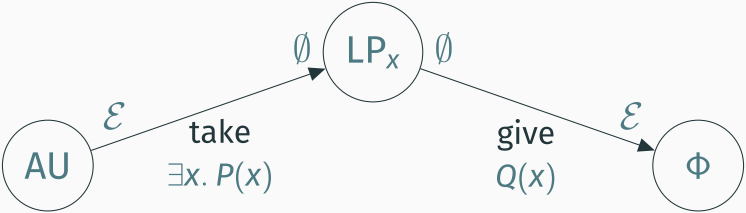

Let us prove

$$\mathsf{IsCtr}(\ell, \gamma) \vdash$$
$$\langle v.\ \mathsf{CtrV}(\gamma, v) \rangle\ \mathtt{inc}(\ell)\ \langle \mathsf{CtrV}(\gamma, v+1) \rangle$$

Let us prove

$$\boxed{\exists v.\, \ell \mapsto v * \boxed{\bullet\, v}^{\gamma}}^{\mathcal{N}} \vdash$$

$$\left\langle v.\, \boxed{\circ\, v}^{\gamma} \right\rangle \texttt{inc}(\ell) \left\langle \boxed{\circ\, v + 1}^{\gamma} \right\rangle^{\top \setminus \mathcal{N}}$$

Let us prove

$$\boxed{\exists v.\, \ell \mapsto v * \boxed{\bullet\, v}^{\gamma}}^{\mathcal{N}} \vdash$$

$$\langle v.\ \boxed{\circ\, v}^{\gamma} \rangle\ \texttt{inc}(\ell)\ \langle \boxed{\circ\, v + 1}^{\gamma} \rangle^{\top \setminus \mathcal{N}}$$

$\mathsf{AU} \triangleq {}^{\top\setminus\mathcal{N}}\Rrightarrow^{\emptyset} \exists w.\ \boxed{\circ\, w}^{\gamma} * \mathsf{LP}_w \quad \mathsf{LP}_w \triangleq \boxed{\circ\, w+1}^{\gamma}\ {}^{\emptyset}\!\Rrightarrow\!\!*^{\top\setminus\mathcal{N}}\ \Phi$

Context: $\boxed{\exists w.\ \ell \mapsto w * \boxed{\bullet\, w}^{\gamma}}^{\mathcal{N}}$

$\{\mathsf{AU}\}_{\top}$

$\mathtt{inc}(\ell)$

$\{\Phi\}_{\top}$

$\mathsf{AU} \triangleq {}^{\top\setminus\mathcal{N}}\!\Rrightarrow^{\emptyset} \exists w.\, \boxed{\circ\, w}^{\gamma} * \mathsf{LP}_w \quad \mathsf{LP}_w \triangleq \boxed{\circ\, w+1}^{\gamma}\; {}^{\emptyset}\!\Rrightarrow\!\!*^{\top\setminus\mathcal{N}}\, \Phi$

Context: $\boxed{\exists w.\, \ell \mapsto w * \boxed{\bullet\, w}^{\gamma}}^{\mathcal{N}}$

$\{\mathsf{AU}\}_{\top}$

$\texttt{let}\ v = !\ell;$

$\texttt{CAS}(\ell, v, v+1)$ (success case)

$\mathsf{AU} \triangleq {}^{\top\setminus\mathcal{N}}\!\Rrightarrow^{\emptyset} \exists w.\, \boxed{\circ\, w}^{\gamma} * \mathsf{LP}_w \quad \mathsf{LP}_w \triangleq \boxed{\circ\, w+1}^{\gamma}\; {}^{\emptyset}\!\Rrightarrow\!\!*^{\top\setminus\mathcal{N}}\, \Phi$

Context: $\boxed{\exists w.\, \ell \mapsto w * \boxed{\bullet\, w}^{\gamma}}^{\mathcal{N}}$

$\{\mathsf{AU}\}_{\top}$

$\{\ell \mapsto w * \boxed{\bullet\, w}^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$

$\texttt{let}\; v = {!\ell};$

$\{\mathsf{AU}\}_{\top}$

$\texttt{CAS}(\ell, v, v+1)$ (success case)

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Context: $\boxed{\exists w.\ \ell \mapsto w * \boxed{\bullet\, w}^{\gamma}}^{\mathcal{N}}$

$\{\mathsf{AU}\}_{\top}$

$\{\ell \mapsto w * \boxed{\bullet\, w}^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$

$\texttt{CAS}(\ell, v, v+1)$ (success case)

$\mathsf{AU} \triangleq {}^{\top\setminus\mathcal{N}}\!\Rrightarrow^{\emptyset} \exists w.\ [\circ\, w]^{\gamma} * \mathsf{LP}_w \quad \mathsf{LP}_w \triangleq [\circ\, w+1]^{\gamma}\ {}^{\emptyset}\!\Rrightarrow^{\top\setminus\mathcal{N}} \Phi$

Context: $\boxed{\exists w.\ \ell \mapsto w * [\bullet\, w]^{\gamma}}^{\mathcal{N}}$

$\{\mathsf{AU}\}_{\top}$

$\{\ell \mapsto w * [\bullet\, w]^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$

$\mathtt{CAS}(\ell, v, v+1)$ (success case)

$\{\ell \mapsto v+1 * [\bullet\, v]^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$

$\mathsf{AU} \triangleq {}^{\top\setminus\mathcal{N}}\Rrightarrow^{\emptyset} \exists w.\, \boxed{\circ\, w}^{\gamma} * \mathsf{LP}_w \quad \mathsf{LP}_w \triangleq \boxed{\circ\, w+1}^{\gamma}\, {}^{\emptyset}\!\!\Rrightarrow\!\!\ast^{\top\setminus\mathcal{N}}\, \Phi$

Context: $\boxed{\exists w.\, \ell \mapsto w * \boxed{\bullet\, w}^{\gamma}}^{\mathcal{N}}$

$\{\mathsf{AU}\}_{\top}$

$\{\ell \mapsto w * \boxed{\bullet\, w}^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$

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Context: $\fbox{$\exists w.\, \ell \mapsto w * \boxed{\bullet\, w}^{\gamma}$}^{\mathcal{N}}$

$\{\mathsf{AU}\}_{\top}$

$\quad \{\ell \mapsto w * \boxed{\bullet\, w}^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$

$\quad \mathtt{CAS}(\ell, v, v + 1)$ (success case)

$\quad \{\ell \mapsto v + 1 * \boxed{\bullet\, v}^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$

$\qquad \{\ell \mapsto v + 1 * \boxed{\bullet\, v \cdot \circ\, w}^{\gamma} * \mathsf{LP}_w\}_{\emptyset}$

$$\mathsf{AU} \triangleq {}^{\top\setminus\mathcal{N}}\!\Rrightarrow^{\emptyset} \exists w.\, \boxed{\circ\, w}^{\gamma} * \mathsf{LP}_w \qquad \mathsf{LP}_w \triangleq \boxed{\circ\, w + 1}^{\gamma}\; {}^{\emptyset}\!\Rrightarrow\!\!*^{\top\setminus\mathcal{N}}\, \Phi$$

Context: $\boxed{\exists w.\, \ell \mapsto w * \boxed{\bullet\, w}^{\gamma}}^{\mathcal{N}}$

$$\{\mathsf{AU}\}_{\top}$$

$$\{\ell \mapsto w * \boxed{\bullet\, w}^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$$

$\texttt{CAS}(\ell, v, v+1)$ (success case)

$$\{\ell \mapsto v + 1 * \boxed{\bullet\, v}^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$$

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$\mathsf{AU} \triangleq {}^{\top\setminus\mathcal{N}}{\Rrightarrow}^{\emptyset}\, \exists w.\, \boxed{\circ w}^{\gamma} * \mathsf{LP}_w \quad \mathsf{LP}_w \triangleq \boxed{\circ w + 1}^{\gamma}\ {}^{\emptyset}{\Rrightarrow\!\!*}^{\top\setminus\mathcal{N}}\ \Phi$

Context: $\boxed{\exists w.\, \ell \mapsto w * \boxed{\bullet w}^{\gamma}}^{\mathcal{N}}$

$\{\mathsf{AU}\}_{\top}$

$\{\ell \mapsto w * \boxed{\bullet w}^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$

$\mathtt{CAS}(\ell, v, v+1)$ (success case)

$\{\ell \mapsto v + 1 * \boxed{\bullet v}^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$

$\{\ell \mapsto v + 1 * \boxed{\bullet v \cdot \circ v}^{\gamma} * \mathsf{LP}_v\}_{\emptyset}$

$\mathrm{AU} \triangleq {}^{\top\setminus\mathcal{N}}\Rrightarrow^{\emptyset} \exists w.\, \boxed{\circ\, w}^{\gamma} * \mathrm{LP}_w \quad \mathrm{LP}_w \triangleq \boxed{\circ\, w + 1}^{\gamma}\; {}^{\emptyset}\!\!\Rrightarrow\!\!*^{\top\setminus\mathcal{N}}\, \Phi$

Context: $\boxed{\exists w.\, \ell \mapsto w * \boxed{\bullet\, w}^{\gamma}}^{\mathcal{N}}$

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$\quad \{\ell \mapsto w * \boxed{\bullet\, w}^{\gamma} * \mathrm{AU}\}_{\top\setminus\mathcal{N}}$

$\quad \texttt{CAS}(\ell, v, v+1)$ (success case)

$\quad \{\ell \mapsto v + 1 * \boxed{\bullet\, v}^{\gamma} * \mathrm{AU}\}_{\top\setminus\mathcal{N}}$

$\qquad \{\ell \mapsto v + 1 * \boxed{\bullet\, v \cdot \circ\, v}^{\gamma} * \mathrm{LP}_v\}_{\emptyset}$

$\mathrm{AU} \triangleq {}^{\top\setminus\mathcal{N}}\Rrightarrow^{\emptyset} \exists w.\, \boxed{\circ w}^{\gamma} * \mathrm{LP}_w \quad \mathrm{LP}_w \triangleq \boxed{\circ w + 1}^{\gamma}\ {}^{\emptyset}\!\!\Rrightarrow\!\!\ast^{\top\setminus\mathcal{N}}\ \Phi$

Context: $\boxed{\exists w.\, \ell \mapsto w * \boxed{\bullet w}^{\gamma}}^{\mathcal{N}}$

$$
\begin{aligned}
&\{\mathrm{AU}\}_{\top} \\
&\quad \{\ell \mapsto w * \boxed{\bullet w}^{\gamma} * \mathrm{AU}\}_{\top\setminus\mathcal{N}} \\
&\quad \texttt{CAS}(\ell, v, v+1) \qquad \text{(success case)} \\
&\quad \{\ell \mapsto v + 1 * \boxed{\bullet v}^{\gamma} * \mathrm{AU}\}_{\top\setminus\mathcal{N}} \\
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&\qquad \{\ell \mapsto v + 1 * \boxed{\bullet v + 1 \cdot \circ v + 1}^{\gamma} * \mathrm{LP}_v\}_{\emptyset}
\end{aligned}
$$

$\mathsf{AU} \triangleq {}^{\top\setminus\mathcal{N}}\!\!\Rrightarrow^{\emptyset} \exists w.\, \boxed{\circ\, w}^{\gamma} * \mathsf{LP}_w \qquad \mathsf{LP}_w \triangleq \boxed{\circ\, w+1}^{\gamma}\; {}^{\emptyset}\!\!\Rrightarrow\!\!*^{\top\setminus\mathcal{N}}\, \Phi$

Context: $\boxed{\exists w.\, \ell \mapsto w * \boxed{\bullet\, w}^{\gamma}}^{\mathcal{N}}$

$\{\mathsf{AU}\}_{\top}$

$\{\ell \mapsto w * \boxed{\bullet\, w}^{\gamma} * \mathsf{AU}\}_{\top\setminus\mathcal{N}}$

$\texttt{CAS}(\ell, v, v+1)$ (success case)

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$$
\begin{array}{l}
\{\mathsf{AU}\}_{\top} \\
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\qquad \{\ell \mapsto v+1 * \boxed{\bullet v+1 \cdot \circ v+1}^{\gamma} * \mathsf{LP}_v\}_{\emptyset} \\
\quad \{\ell \mapsto v+1 * \boxed{\bullet v+1}^{\gamma} * \Phi\}_{\top\setminus\mathcal{N}} \\
\{\Phi\}_{\top}
\end{array}
$$

$\mathsf{AU} \triangleq {}^{\top\setminus\mathcal{N}}{\Rrightarrow}^{\emptyset}\, \exists w.\, \boxed{\circ\, w}^{\gamma} * \mathsf{LP}_w \qquad \mathsf{LP}_w \triangleq \boxed{\circ\, w + 1}^{\gamma}\; {}^{\emptyset}{\Rrightarrow}^{\top\setminus\mathcal{N}}\, \Phi$

Context: $\boxed{\exists w.\, \ell \mapsto w * \boxed{\bullet\, w}^{\gamma}}^{\mathcal{N}}$

$\{\mathsf{AU}\}_{\top}$

We now have

$$\mathsf{IsCtr}(\ell, \gamma) \vdash$$
$$\langle v.\, \mathsf{CtrV}(\gamma, v)\rangle\ \mathtt{inc}(\ell)\ \langle \mathsf{CtrV}(\gamma, v+1)\rangle$$

$\{\Phi\}_{\top}$

$\mathsf{AU} \triangleq {}^{\top\setminus\mathcal{N}}\!\Rrightarrow^{\emptyset} \exists w.\, \boxed{\circ\, w}^{\gamma} * \mathsf{LP}_w \quad \mathsf{LP}_w \triangleq \boxed{\circ\, w+1}^{\gamma}\; {}^{\emptyset}\!\Rrightarrow\!\!*^{\top\setminus\mathcal{N}}\, \Phi$

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$$\langle v.\, \mathsf{CtrV}(\gamma, v)\rangle\ \texttt{inc}(\ell)\ \langle \mathsf{CtrV}(\gamma, v+1)\rangle$$

What about the invariant rule?

$\{\Phi\}_{\top}$

Invariant rule

Given: $\langle P \rangle\ e\ \langle Q \rangle^{\top}$

$$\frac{\langle x.\, P * I \rangle\ e\ \langle Q * I \rangle^{\mathcal{E}} \qquad \mathcal{N} \subseteq \mathcal{E}}{\boxed{I}^{\mathcal{N}} \vdash \langle x.\, P \rangle\ e\ \langle Q \rangle^{\mathcal{E} \setminus \mathcal{N}}}$$

Invariant rule

Given: $\langle P \rangle\, e\, \langle Q \rangle^{\top}$

Show: $\boxed{P * \ldots \vee Q * \ldots}^{\mathcal{N}} \vdash \{\ldots\}\, e\, \{\ldots\}$

Invariant rule

Given: $\langle P \rangle\ e\ \langle P \rangle^{\top}$

Show: $\boxed{P}^{\mathcal{N}} \vdash \{\mathrm{True}\}\ e\ \{\mathrm{True}\}$

Invariant rule

Given: $\forall \Phi.\ \left({}^{\top}{\models\!\!\!\Rrightarrow}^{\emptyset} P * (P \mathrel{-\!\!*} {}^{\emptyset}{\models\!\!\!\Rrightarrow}^{\top} \Phi) \right) \mathrel{-\!\!*} \mathrm{wp}\ e\ \{\Phi\}$

Show: $\boxed{P}^{\mathcal{N}} \vdash \{\mathrm{True}\}\ e\ \{\mathrm{True}\}$

Invariant rule

Given: $\forall \Phi.\ \left({}^{\top}\!\!\Rrightarrow^{\emptyset} P * (P \twoheadrightarrow {}^{\emptyset}\!\!\Rrightarrow^{\top} \Phi) \right) \twoheadrightarrow \mathsf{wp}\ e\ \{\Phi\}$

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Show: $\boxed{P}^{\mathcal{N}} \vdash \mathrm{wp}\ e\ \{\mathrm{True}\}$

It suffices to show:

$$\boxed{P}^{\mathcal{N}} \vdash {}^{\top}\!\!\Rrightarrow^{\emptyset} P * (P \twoheadrightarrow {}^{\emptyset}\!\!\Rrightarrow^{\top} \mathrm{True})$$

Invariant rule

Given: $\forall\Phi.\ \left({}^{\top}{\Rrightarrow}^{\emptyset} P * (P -\!\!* {}^{\emptyset}{\Rrightarrow}^{\top} \Phi)\right) -\!\!* \mathsf{wp}\ e\ \{\Phi\}$

Show: $\boxed{P}^{\mathcal{N}} \vdash \mathsf{wp}\ e\ \{\mathsf{True}\}$

It suffices to show:

$$\boxed{P}^{\mathcal{N}} \vdash {}^{\top}{\Rrightarrow}^{\emptyset} P * (P -\!\!* {}^{\emptyset}{\Rrightarrow}^{\top} \mathsf{True})$$

This is (almost) the invariant accessor!

Invariant rule

Given: $\forall \Phi.\ \left({}^{\top}\!\Rrightarrow^{\emptyset} P * (P -\!\!* \ {}^{\emptyset}\!\Rrightarrow^{\top} \Phi) \right) -\!\!* \ \mathrm{wp}\ e\ \{\Phi\}$

$$\frac{\langle x.\, P * I \rangle\ e\ \langle Q * I \rangle^{\mathcal{E}} \qquad \mathcal{N} \subseteq \mathcal{E}}{\boxed{I}^{\mathcal{N}} \vdash \langle x.\, P \rangle\ e\ \langle Q \rangle^{\mathcal{E} \setminus \mathcal{N}}}$$

$$\boxed{P}^{\mathcal{N}} \vdash {}^{\top}\!\Rrightarrow^{\emptyset} P * (P -\!\!* \ {}^{\emptyset}\!\Rrightarrow^{\top} \mathrm{True})$$

This is (almost) the invariant accessor!

Logically atomic Hoare triples

1. Define $\langle x.\, P \rangle\ e\ \langle Q \rangle$
2. Prove $\langle v.\, \mathsf{CtrV}(\gamma, v) \rangle\ \mathtt{inc}(\ell)\ \langle \mathsf{CtrV}(\gamma, v+1) \rangle$
3. Prove

$$\frac{\langle x.\, P * I \rangle\ e\ \langle Q * I \rangle}{\boxed{I} \vdash \langle x.\, P \rangle\ e\ \langle Q \rangle}$$

Goals achieved... for weaker spec

Logically atomic Hoare triples

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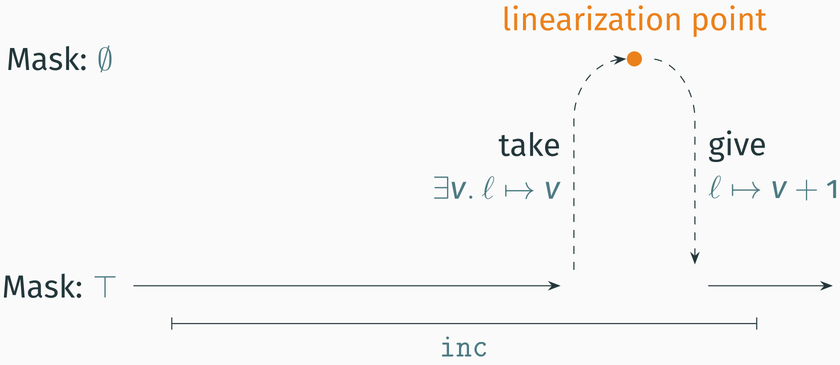
Goals achieved... for weaker spec

What about $\langle v.\, \ell \mapsto v\rangle\; \texttt{inc}(\ell)\; \langle \ell \mapsto v+1\rangle$?

$$\langle v.\, \ell \mapsto v \rangle\ \mathtt{inc}(\ell)\ \langle \ell \mapsto v + 1 \rangle^{\top}$$



$\langle v.\, \ell \mapsto v \rangle\ \texttt{inc}(\ell)\ \langle \ell \mapsto v + 1 \rangle^{\top}$

We can only use the atomic update **once**, at the **linearization point**!

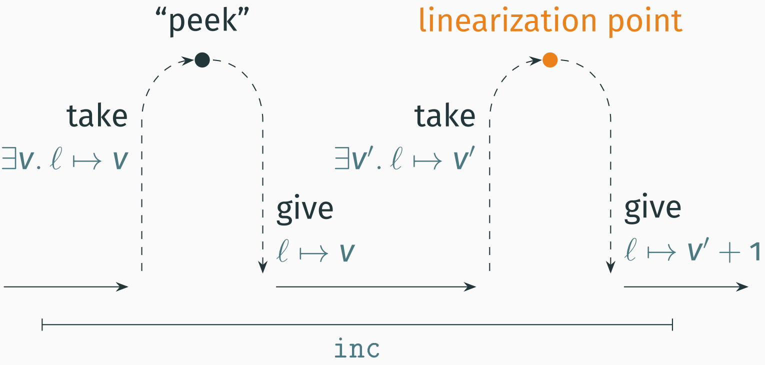
$$
\begin{aligned}
\texttt{rec}\ \texttt{inc}(x) = {} & \texttt{let}\ v = {!x};\\
& \texttt{if}\ \texttt{CAS}(x, v, v + 1)\ \texttt{then}\ v\\
& \texttt{else}\ \texttt{inc}(x)
\end{aligned}
$$

inc

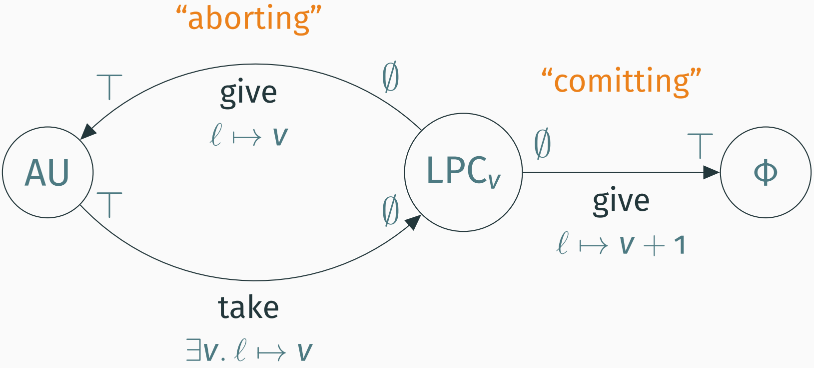
Logical Atomicity, v0.2: aborting

(the Good)

$$\langle v.\, \ell \mapsto v \rangle\ \texttt{inc}(\ell)\ \langle \ell \mapsto v + 1 \rangle^{\top}$$

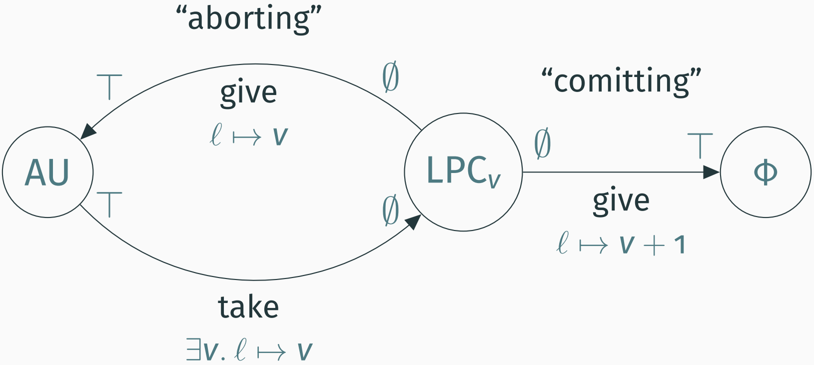


$$\langle \mathbf{v}.\, \ell \mapsto \mathbf{v} \rangle\ \texttt{inc}(\ell)\ \langle \ell \mapsto \mathbf{v} + 1 \rangle^{\top}$$



$$\mathrm{AU} \triangleq {}^{\top}\!\Rrightarrow^{\emptyset} \exists v.\, \ell \mapsto v * \mathrm{LPC}_v$$

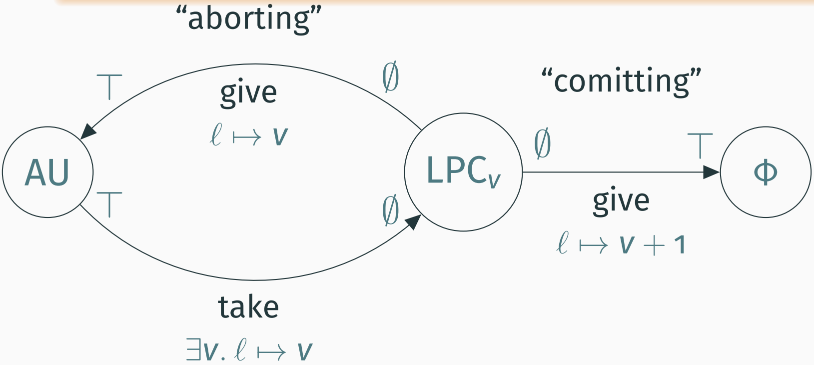
$$\mathrm{LPC}_v \triangleq \left(\ell \mapsto v + 1 \mathrel{-\!\!*} {}^{\emptyset}\!\Rrightarrow^{\top} \Phi\right)$$



$$\mathsf{AU} \triangleq {}^{\top}\!\Rrightarrow^{\emptyset} \exists v.\, \ell \mapsto v * \mathsf{LPC}_v$$

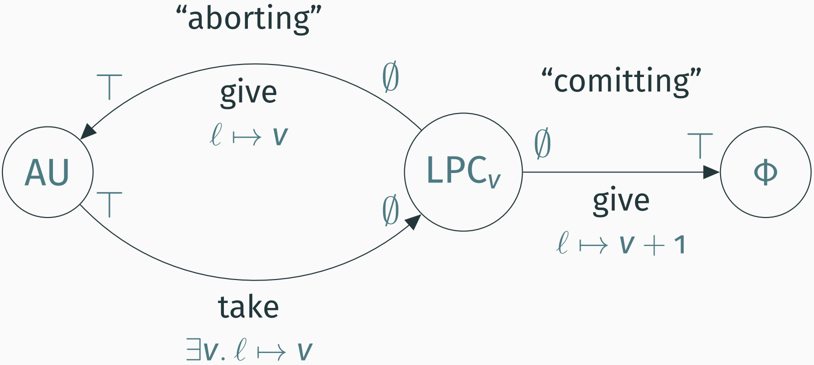
$$\mathsf{LPC}_v \triangleq \quad \wedge\left(\ell \mapsto v + 1 \mathrel{-\!\!*} {}^{\emptyset}\!\Rrightarrow^{\top} \Phi\right)$$

"Additive" conjunction ≅ Internal choice



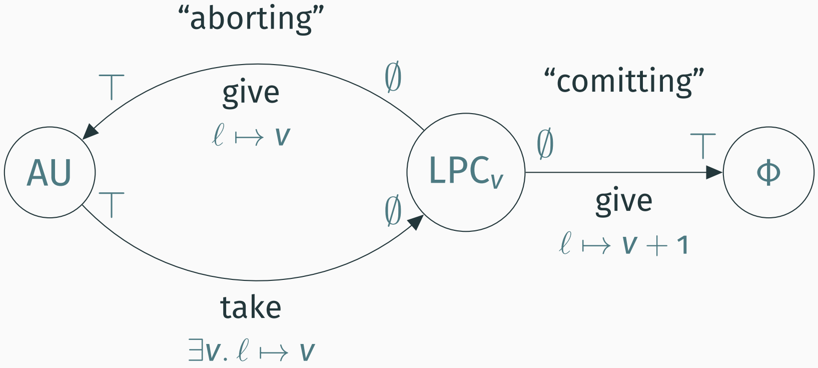
$$\mathrm{AU} \triangleq {}^{\top}{\models\!\!\!\Rrightarrow}^{\emptyset} \exists v.\, \ell \mapsto v * \mathrm{LPC}_v$$

$$\mathrm{LPC}_v \triangleq \left(\ell \mapsto v \mathrel{-\!\!*} {}^{\emptyset}{\models\!\!\!\Rrightarrow}^{\top} \mathrm{AU}\right) \wedge \left(\ell \mapsto v + 1 \mathrel{-\!\!*} {}^{\emptyset}{\models\!\!\!\Rrightarrow}^{\top} \Phi\right)$$



$$\mathsf{AU} \triangleq {}^{\top}\!\!\Rrightarrow^{\emptyset} \exists v.\, \ell \mapsto v * \mathsf{LPC}_v$$

$$\mathsf{LPC}_v \triangleq \left(\ell \mapsto v \mathrel{-\!\!*} {}^{\emptyset}\!\!\Rrightarrow^{\top} \mathsf{AU}\right) \land \left(\ell \mapsto v + 1 \mathrel{-\!\!*} {}^{\emptyset}\!\!\Rrightarrow^{\top} \Phi\right)$$



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3. Prove

$$\frac{\langle x.\, P * I\rangle\; e\; \langle Q * I\rangle}{\boxed{I} \vdash \langle x.\, P\rangle\; e\; \langle Q\rangle}$$

4. Profit!

Logically atomic Hoare triples

1. Define $\langle x.\, P \rangle\; e\; \langle Q \rangle$
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4. ~~Profit!~~ Publish!

Related work: HOCAP

$$\frac{\forall X.\ \mathsf{x}_{\mathsf{cont}} \overset{1/2}{\longmapsto} X * P \sqsubseteq \mathsf{x}_{\mathsf{cont}} \overset{1/2}{\longmapsto} X \cup \{y\} * Q}{\{\mathsf{bag}(x) * P\}x.\mathsf{Push}(y)\{\mathsf{bag}(x) * Q\}}$$

Related work: HOCAP

$$\frac{\forall X.\ \mathsf{x}_{\mathsf{cont}} \overset{1/2}{\longmapsto} X * P \sqsubseteq \mathsf{x}_{\mathsf{cont}} \overset{1/2}{\longmapsto} X \cup \{y\} * Q}{\{\mathsf{bag}(x) * P\}x.\mathsf{Push}(y)\{\mathsf{bag}(x) * Q\}}$$

$$\mathsf{IsCtr}(\ell, \gamma) \vdash \left(\forall v.\ \boxed{\bullet v}^{\gamma} \Rrightarrow\!\!*_{\top\setminus\mathcal{N}} \boxed{\bullet v + 1}^{\gamma} * \Phi\right) \mathrel{-\!\!*} \mathsf{wp}_{\top}\ \mathtt{inc}(\ell)\ \{\Phi\}$$

Related work: HOCAP

Iris-style logically atomic spec:

$$\mathsf{IsCtr}(\ell, \gamma) \vdash \langle v.\, \mathsf{CtrV}(\gamma, v) \rangle\ \mathtt{inc}(\ell)\ \langle \mathsf{CtrV}(\gamma, v + 1) \rangle^{\top \setminus \mathcal{N}}$$

$$\boxed{\circ\, v}^{\gamma}$$

HOCAP-style logically atomic spec:

$$\mathsf{IsCtr}(\ell, \gamma) \vdash \left(\forall v.\, \boxed{\bullet\, v}^{\gamma} \Rrightarrow\!\!*_{\top \setminus \mathcal{N}} \boxed{\bullet\, v + 1}^{\gamma} * \Phi \right) -\!\!* \\ \mathsf{wp}_{\top}\ \mathtt{inc}(\ell)\ \{\Phi\}$$

Related work: HOCAP

Iris-style logically atomic spec:

$$\langle v.\, \ell \mapsto v \rangle \; \texttt{inc}(\ell) \; \langle \ell \mapsto v + 1 \rangle^{\top}$$

HOCAP-style logically atomic spec:

???

Related work: HOCAP

Iris-style logically atomic spec:

$$\langle v.\, \ell \mapsto v \rangle\ \texttt{inc}(\ell)\ \langle \ell \mapsto v + 1 \rangle^{\top}$$

HOC

HOCAP-style logical atomicity is a pattern, not an abstraction—it is unclear how to even state the invariant rule.

Related work: TaDA

- has invariant rule, aborting and arbitrary pre-/postconditions

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- has invariant rule, aborting and arbitrary pre-/postconditions

Open region rule

$$\frac{\lambda; \mathcal{A} \vdash \mathbb{V}x \in X. \left\langle p_p \,\middle|\, I(\mathbf{t}_a^\lambda(x)) * p(x) \right\rangle \ \mathbb{C} \quad \exists\!\!\exists y \in Y. \left\langle q_p(x,y) \,\middle|\, I(\mathbf{t}_a^\lambda(x)) * q(x,y) \right\rangle}{\lambda + 1; \mathcal{A} \vdash \mathbb{V}x \in X. \left\langle p_p \,\middle|\, \mathbf{t}_a^\lambda(x) * p(x) \right\rangle \ \mathbb{C} \quad \exists\!\!\exists y \in Y. \left\langle q_p(x,y) \,\middle|\, \mathbf{t}_a^\lambda(x) * q(x,y) \right\rangle}$$

Related work: TaDA

- has invariant rule, aborting and arbitrary pre-/postconditions
- ties atomicity to level of abstraction

Open region rule

$$\frac{\lambda; \mathcal{A} \vdash \mathbb{A} x \in X. \left\langle p_p \,\middle|\, I(\mathbf{t}_a^\lambda(x)) * p(x) \right\rangle \; \mathbb{C} \quad \mathbb{E} y \in Y. \left\langle q_p(x, y) \,\middle|\, I(\mathbf{t}_a^\lambda(x)) * q(x, y) \right\rangle}{\lambda + 1; \mathcal{A} \vdash \mathbb{A} x \in X. \left\langle p_p \,\middle|\, \mathbf{t}_a^\lambda(x) * p(x) \right\rangle \; \mathbb{C} \quad \mathbb{E} y \in Y. \left\langle q_p(x, y) \,\middle|\, \mathbf{t}_a^\lambda(x) * q(x, y) \right\rangle}$$

Related work: TaDA

- has invariant rule, aborting and arbitrary pre-/postconditions
- ties atomicity to level of abstraction

TaDA cannot prove

$$\langle v.\, \ell \mapsto v \rangle\ \texttt{inc}(\ell)\ \langle \ell \mapsto v + 1 \rangle^{\top}$$

as there is no abstraction!

Logically Atomic Case Studies

- Increment

Logically Atomic Case Studies

- Increment on abstract heap

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- Increment on abstract heap
- Elimination Stack on abstract heap

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Logically Atomic Case Studies

- Increment on abstract heap
- Elimination Stack on abstract heap
- F
- A
- R … v)
- Herlihy-Wing-Queue (by Rodolphe, Derek, Gaurav)

Many of these use helping, which logical atomicity v0.2 does not support!

Logical Atomicity, v1: laters

(the Ugly)

Helping occurs when one threads' linearization point is executed by another thread.

Helping occurs when one threads' linearization point is executed by another thread.

$\boxed{\ldots * \mathsf{AU} * \ldots}$

Helping occurs when one threads' linearization point is executed by another thread.

▷ AU is useless!

... ∗ AU ∗ ...

Laterability

A proposition P is laterable if it can be split into "something persistent" and "something later"

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A proposition P is laterable if it can be split into "something persistent" and "something later"

$$\text{create:} \quad \triangleright I \Rrightarrow\!\!\!*_{\mathcal{E}} \boxed{I}^{\mathcal{N}}$$

$$\text{open:} \quad \boxed{I}^{\mathcal{N}} \quad {}^{\mathcal{N}}\!\Rrightarrow\!\!\!*^{\top} \triangleright I$$

$$\text{close:} \quad \boxed{I}^{\mathcal{N}} * \triangleright I \; {}^{\top}\!\Rrightarrow\!\!\!*^{\mathcal{N}}$$

Laterability

A proposition P is laterable if it can be split into "something persistent" and "something later"

Laterable assertions P can be losslessly put into an invariant:

$$P \Rrightarrow \exists Q. \boxed{Q} * (\triangleright Q \Rrightarrow P)$$

Laterability

A proposition P is laterable if it can be split into "something persistent" and "something later":

$$\text{laterable}(P) \triangleq P \twoheadrightarrow\!\!* \; \exists Q.\ \triangleright Q * \Box(\triangleright Q \twoheadrightarrow\!\!* \diamond P)$$

Laterable assertions P can be losslessly put into an invariant:

$$P \Rrightarrow \exists Q.\ \boxed{Q} * (\triangleright Q \Rrightarrow P)$$

$$\text{laterable}(\triangleright P) \qquad \frac{\text{timeless}(P)}{\text{laterable}(P)} \qquad \frac{\text{persistent}(P)}{\text{laterable}(P)}$$

$$\text{laterable}(\triangleright P) \qquad \frac{\text{timeless}(P)}{\text{laterable}(P)} \qquad \frac{\text{persistent}(P)}{\text{laterable}(P)}$$

$$\frac{\text{laterable}(P) \qquad \text{laterable}(Q)}{\text{laterable}(P * Q)}$$

timeless(P) persistent(P)

Needed for helping:

laterable(AU)

$$\text{laterable}(\text{make\_laterable}(P))$$

$$\text{make\_laterable}(P) \vdash P$$

$$\text{laterable}(\text{make\_laterable}(P))$$

$$\text{make\_laterable}(P) \vdash P$$

$$\frac{\text{laterable}(\Gamma) \qquad \Diamond\,\Gamma \vdash P}{\Gamma \vdash \text{make\_laterable}(P)}$$

$$\text{laterable}(\text{make\_laterable}(P))$$

$$\text{make\_laterable}(P) \triangleq$$
$$\exists Q.\ \triangleright Q * \Box(\triangleright Q -\!\!* P)$$

$$\Gamma \vdash \text{make\_laterable}(P)$$

Defining logically atomic triples (v1)

$$\langle x.\, P(x)\rangle\; e\; \langle v.\, Q(x,v)\rangle^{\mathcal{E}} \triangleq \forall \Phi.\, \mathsf{AU} \twoheadrightarrow* \mathsf{wp}_{\top}\, e\, \{\Phi\}$$

$$\mathsf{AU} \triangleq \nu U.\, \mathsf{make\_laterable}\Big({}^{\mathcal{E}}{\Rrightarrow}^{\emptyset}\, \exists x.\, P(x) * \\ \big(\big(P(x)\; {}^{\emptyset}{\equiv\!\!*}^{\mathcal{E}}\, U\big) \wedge \big(\forall v.\, Q(x,v)\; {}^{\emptyset}{\equiv\!\!*}^{\mathcal{E}}\, \Phi(v)\big)\big)\Big)$$

Defining logically atomic triples (v1)

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$$\mathsf{AU} \triangleq \nu U.\, \mathsf{make\_laterable}\Big({}^{\mathcal{E}}{\Rrightarrow}^{\emptyset}\, \exists x.\, P(x) * \Big(\big(P(x) \;{}^{\emptyset}{\equiv\!\!*}^{\mathcal{E}}\; U\big) \wedge \big(\forall v.\, Q(x,v) \;{}^{\emptyset}{\equiv\!\!*}^{\mathcal{E}}\; \Phi(v)\big)\Big)\Big)$$

Defining logically atomic triples (v1)

$$\langle x.\, P(x) \rangle\ e\ \langle v.\, Q(x,v) \rangle^{\mathcal{E}} \triangleq \forall \Phi.\, \mathsf{AU} \mathrel{-\!\!*} \mathsf{wp}_{\top}\ e\ \{\Phi\}$$

$$\mathsf{AU} \triangleq \nu U.\, \mathsf{make\_laterable}\Big({}^{\mathcal{E}}\!\!\Rrightarrow^{\emptyset} \exists x.\, P(x) * \Big(\big(P(x) \;{}^{\emptyset}\!\!\Rrightarrow\!\!*^{\mathcal{E}}\; U\big) \wedge \big(\forall v.\, Q(x,v) \;{}^{\emptyset}\!\!\Rrightarrow\!\!*^{\mathcal{E}}\; \Phi(v)\big) \Big) \Big)$$

Defining logically atomic triples (v1)

$$\langle x.\, P(x)\rangle\; e\; \langle v.\, Q(x,v)\rangle^{\mathcal{E}} \triangleq \forall \Phi.\, \mathsf{AU} \mathrel{-\!\!*} \mathsf{wp}_{\top}\; e\; \{\Phi\}$$

$$\mathsf{AU} \triangleq \nu U.\, \mathsf{make\_laterable}\Big({}^{\mathcal{E}}\!\Rrightarrow^{\emptyset} \exists x.\, P(x) * \big((P(x) \;{}^{\emptyset}\!\!\Rrightarrow\!\!*^{\mathcal{E}}\, U) \wedge (\forall v.\, Q(x,v) \;{}^{\emptyset}\!\!\Rrightarrow\!\!*^{\mathcal{E}}\, \Phi(v)) \big) \Big)$$

Defining logically atomic triples (v1)

$$\langle x.\, P(x) \rangle \; e \; \langle v.\, Q(x,v) \rangle^{\mathcal{E}} \triangleq \forall \Phi.\, \mathsf{AU} \mathrel{-\!\!*} \mathsf{wp}_{\top}\, e \, \{\Phi\}$$

$$\mathsf{AU} \triangleq \nu U.\, \text{make\_laterable}\Big({}^{\mathcal{E}}\!\!\Rrightarrow^{\emptyset} \exists x.\, P(x) * \big((P(x) \; {}^{\emptyset}\!\!\Rrightarrow\!\!*^{\mathcal{E}}\, U) \wedge (\forall v.\, Q(x,v) \; {}^{\emptyset}\!\!\Rrightarrow\!\!*^{\mathcal{E}}\, \Phi(v)) \big) \Big)$$

Laterable atomic updates?

Ugly introduction rule:

$$\frac{\mathsf{laterable}(\Gamma) \qquad {}^{\mathcal{E}}{\models\!\!\!\Rrightarrow}^{\top}\, \exists x.\, P(x) * \Big(\big(P(x) \;{}^{\emptyset}\!\!\Rrightarrow\!\!\ast^{\mathcal{E}}\, \Gamma\big) \wedge \big(\forall v.\, Q(x,v) \;{}^{\emptyset}\!\!\Rrightarrow\!\!\ast^{\mathcal{E}}\, \Phi(v)\big)\Big)}{\Gamma \vdash \mathsf{AU}}$$

Laterable atomic updates?

Ugly introduction rule:

$$\frac{\mathsf{laterable}(\Gamma) \qquad {}^{\mathcal{E}}{\models\!\!\!\Rightarrow}^{\top}\, \exists x.\, P(x) * \Big(\big(P(x)\ {}^{\emptyset}{\equiv\!\!\ast}^{\mathcal{E}}\ \Gamma\big) \land \big(\forall v.\, Q(x, v)\ {}^{\emptyset}{\equiv\!\!\ast}^{\mathcal{E}}\ \Phi(v)\big) \Big)}{\Gamma \vdash \mathsf{AU}}$$

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$$\frac{\mathrm{laterable}(\Gamma)\qquad {}^{\mathcal{E}}{\models\!\!\Rrightarrow}^{\top}\, \exists x.\, P(x) * \Big(\big(P(x)\ {}^{\emptyset}{\Rrightarrow\!\!*}^{\mathcal{E}}\ \Gamma\big) \land \big(\forall v.\, Q(x,v)\ {}^{\emptyset}{\Rrightarrow\!\!*}^{\mathcal{E}}\ \Phi(v)\big) \Big)}{\Gamma \vdash \mathrm{AU}}$$

Laterable atomic updates!

We can now do helping:

$$\mathsf{AU} \Rrightarrow\!\!\ast\; \exists Q.\ \triangleright Q * (\triangleright Q \Rrightarrow \mathsf{AU})$$

$$\Rrightarrow\!\!\ast\; \boxed{\ldots * Q * \ldots} * (\triangleright Q \Rrightarrow \mathsf{AU})$$

Logical Atomicity lets us give

- concise and powerful
- Hoare-style specifications
- to concurrent data structures
- that make use of helping.