Iris: Monoids and Invariants as an Orthogonal Basis for Concurrent Reasoning

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Iris

A new separation logic that

- can verify complex, lock-free concurrent datastructures
- permits modular (thread-local) reasoning

Tons of prior program logics

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- CSL [O'H07]

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- RGSep [VP07]

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- CaReSL [TDB13]

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- iCAP [SB14]

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Do we really need yet another concurrency logic?

Yet another concurrency logic

Iris addresses two problems

- Simplifying the foundations of concurrent reasoning
- Supporting a notion of *logical atomicity*

Problem 1: Protocols

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Problem 1: Protocols

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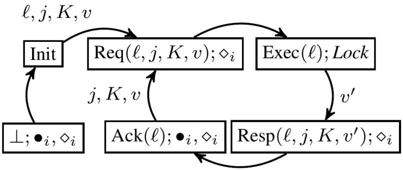
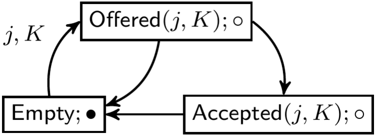
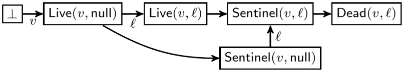
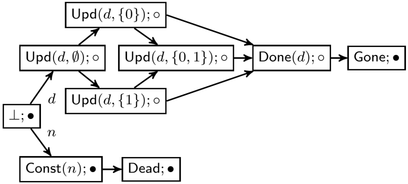
Common approach: "protocol" to deal with interference

STSs in CaReSL









Complex rules built-in as primitives

CaReSL: $$\frac{\mathcal{C} \vdash \forall b \overset{\text{rely}}{\sqsupseteq_\pi} b_0.\ (\!|\pi[\![b]\!] * P|\!)\ i \Rrightarrow_{\mathrm{I}} a\ (\!|x.\ \exists b' \overset{\text{guar}}{\sqsupseteq_\pi} b.\ \pi[\![b']\!] * Q|\!)}{\mathcal{C} \vdash \left\{ \boxed{b_0}^{n}_{\pi} * \triangleright P \right\}\ i \Rrightarrow a\ \left\{ x.\ \exists b'.\ \boxed{b'}^{n}_{\pi} * Q \right\}} \textsc{UpdIsl}$$

Complex rules built-in as primitives

CaReSL:

$$\frac{\mathcal{C} \vdash \forall b \overset{\text{rely}}{\sqsupseteq_\pi} b_0.\ (\!|\pi[\![b]\!] * P|\!)\ i \Rrightarrow_{\mathrm{I}} a\ (\!|x.\ \exists b' \overset{\text{guar}}{\sqsupseteq_\pi} b.\ \pi[\![b']\!] * Q|\!)}{\mathcal{C} \vdash \left\{ \boxed{b_0}^{n}_{\pi} * \triangleright P \right\}\ i \Rrightarrow a\ \left\{ x.\ \exists b'.\ \boxed{b'}^{n}_{\pi} * Q \right\}}\ \textsc{UpdIsl}$$

iCAP:

$$\frac{\begin{array}{c}\Gamma, \Delta \mid \Phi \vdash \mathsf{stable}(\mathsf{P}) \qquad \Gamma, \Delta \mid \Phi \vdash \forall y.\ \mathsf{stable}(\mathsf{Q}(y)) \\ \Gamma, \Delta \mid \Phi \vdash n \in C \qquad \Gamma, \Delta \mid \Phi \vdash \forall x \in X.\ (x, f(x)) \in \overline{T(A)} \vee f(x) = x \\ \Gamma \mid \Phi \vdash \forall x \in X.\ (\Delta).\langle \mathsf{P} * \circledast_{\alpha \in A}[\alpha]^{\mathsf{n}}_{g(\alpha)} * \triangleright I(x) \rangle\ c\ \langle \mathsf{Q}(x) * \triangleright I(f(x)) \rangle^{C \setminus \{n\}}\end{array}}{\begin{array}{c}\Gamma \mid \Phi \vdash (\Delta).\ \langle \mathsf{P} * \circledast_{\alpha \in A}[\alpha]^{\mathsf{n}}_{g(\alpha)} * \mathsf{region}(X, T, I, n) \rangle \\ c \\ \langle \exists x.\ \mathsf{Q}(x) * \mathsf{region}(\{f(x)\}, T, I, n) \rangle^{C}\end{array}}\ \textsc{Atomic}$$

TaDA:

Use atomic rule

$$\frac{\begin{array}{c} a \notin \mathcal{A} \quad \forall x \in X.\ (x, f(x)) \in \mathcal{T}_{\mathbf{t}}(\mathrm{G})^{*} \\ \lambda; \mathcal{A} \vdash \mathbb{W} x \in X.\ \langle p_p \mid I(\mathbf{t}^{\lambda}_{a}(x)) * p(x) * [\mathrm{G}]_a \rangle\ \mathbb{C}\ \exists\!\!\exists y \in Y.\ \langle q_p(x, y) \mid I(\mathbf{t}^{\lambda}_{a}(f(x))) * q(x, y) \rangle \end{array}}{\lambda + 1; \mathcal{A} \vdash \mathbb{W} x \in X.\ \langle p_p \mid \mathbf{t}^{\lambda}_{a}(x) * p(x) * [\mathrm{G}]_a \rangle\ \mathbb{C}\ \exists\!\!\exists y \in Y.\ \langle q_p(x, y) \mid \mathbf{t}^{\lambda}_{a}(f(x)) * q(x, y) \rangle}$$

Complex rules built-in as primitives

CaReSL: $$\dfrac{\mathcal{C} \vdash \forall b \overset{\text{rely}}{\sqsupseteq_\pi} b_0.\ \langle\!| \pi[\![b]\!] * P |\!\rangle\ i \Rrightarrow_1 a\ \langle\!| x.\ \exists b' \overset{\text{guar}}{\sqsupseteq_\pi} b.\ \pi[\![b']\!] * Q |\!\rangle}{\mathcal{C} \vdash \left\{ \boxed{b_0}^{n}_{\pi} * \triangleright P \right\}\ i \Rrightarrow a\ \left\{ x.\ \exists b'.\ \boxed{b'}^{n}_{\pi} * Q \right\}}\ \textsc{UpdIsl}$$

All you need are two simple primitives:

- *Monoids* to *express* protocols.
- *Invariants* to *enforce* protocols.

TaDA: Use atomic rule

$$\dfrac{a \notin \mathcal{A} \quad \forall x \in X.\, (x, f(x)) \in \mathcal{T}_{\mathbf{t}}(\mathrm{G})^* \qquad \lambda; \mathcal{A} \vdash \mathbb{W} x \in X.\, \langle p_p \mid I(\mathbf{t}^{\lambda}_{a}(x)) * p(x) * [\mathrm{G}]_a \rangle\ \mathbb{C}\ \exists\!\!\exists y \in Y.\, \langle q_p(x,y) \mid I(\mathbf{t}^{\lambda}_{a}(f(x))) * q(x,y) \rangle}{\lambda + 1; \mathcal{A} \vdash \mathbb{W} x \in X.\, \langle p_p \mid \mathbf{t}^{\lambda}_{a}(x) * p(x) * [\mathrm{G}]_a \rangle\ \mathbb{C}\ \exists\!\!\exists y \in Y.\, \langle q_p(x,y) \mid \mathbf{t}^{\lambda}_{a}(f(x)) * q(x,y) \rangle}$$

Problem 2: Specifying Atomicity

Complex implementation

fn *push_fancy*(s, x) {
  let h_n = **ref** (next $\mapsto$ **null**, value $\mapsto$ x) **in**
  let h_o = $!s$.head **in**
  h_n.next := h_o;
  let b = **cas**(s.head, h_o, h_n) **in**
  if b **then** () **else**
  let o = **ref** (state $\mapsto$ 0, value $\mapsto$ x) **in**
  s.offer := o;
  s.offer := **null**;
  let b = **cas**(o.state, 0, 2) **in**
  if b **then** *push_fancy*(s, x) **else skip**
}

Problem 2: Specifying Atomicity

Complex implementation

$$
\begin{array}{l}
\textbf{fn } \mathit{push\_fancy}(s, x)\ \{ \\
\quad \textbf{let } h_n = \textbf{ref}\ (\mathsf{next} \mapsto \textbf{null}, \mathsf{value} \mapsto x)\ \textbf{in} \\
\quad \textbf{let } h_o = !s.\mathsf{head}\ \textbf{in} \\
\quad h_n.\mathsf{next} := h_o; \\
\quad \textbf{let } b = \textbf{cas}(s.\mathsf{head}, h_o, h_n)\ \textbf{in} \\
\quad \textbf{if } b \textbf{ then } ()\ \textbf{else} \\
\quad \textbf{let } o = \textbf{ref}\ (\mathsf{state} \mapsto 0, \mathsf{value} \mapsto x)\ \textbf{in} \\
\quad s.\mathsf{offer} := o; \\
\quad s.\mathsf{offer} := \textbf{null}; \\
\quad \textbf{let } b = \textbf{cas}(o.\mathsf{state}, 0, 2)\ \textbf{in} \\
\quad \textbf{if } b \textbf{ then } \mathit{push\_fancy}(s, x)\ \textbf{else skip} \\
\}
\end{array}
$$

simple implementation

$$
\begin{array}{l}
\textbf{fn } \mathit{push\_spec}(s, x)\ \{ \\
\quad \textbf{atomic}\ \{ \\
\quad\quad s := (!s) :: x \\
\quad \} \\
\}
\end{array}
$$

Problem 2: Specifying Atomicity

Complex implementation | refines | simple implementation

$$
\begin{array}{l}
\textbf{fn } \mathit{push\_fancy}(s, x) \ \{ \\
\quad \textbf{let } h_n = \textbf{ref } (\text{next} \mapsto \textbf{null}, \text{value} \mapsto x) \textbf{ in} \\
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\quad h_n.\text{next} := h_o; \\
\quad \textbf{let } b = \textbf{cas}(s.\text{head}, h_o, h_n) \textbf{ in} \\
\quad \textbf{if } b \textbf{ then } () \textbf{ else} \\
\quad \textbf{let } o = \textbf{ref } (\text{state} \mapsto 0, \text{value} \mapsto x) \textbf{ in} \\
\quad s.\text{offer} := o; \\
\quad s.\text{offer} := \textbf{null}; \\
\quad \textbf{let } b = \textbf{cas}(o.\text{state}, 0, 2) \textbf{ in} \\
\quad \textbf{if } b \textbf{ then } \mathit{push\_fancy}(s, x) \textbf{ else skip} \\
\}
\end{array}
\quad \leq \quad
\begin{array}{l}
\textbf{fn } \mathit{push\_spec}(s, x) \ \{ \\
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\quad\quad s := (!s) :: x \\
\quad \} \\
\}
\end{array}
$$

Contextual refinement / Linearizability

Problem 2: Specifying Atomicity

Complex implementation | refines | simple implementation

```
fn push_fancy(s, x) {
  let h_n = ref (next ↦ null, value ↦ x) in
  let h_o = !s.head in
  h_n.next := h_o;
  let b = cas(s.head, h_o, h_n) in
  if b then () else
  let o = ref (state ↦ 0, value ↦ x) in
  s.offer := o;
  s.offer := null;
  let b = cas(o.state, 0, 2) in
  if b then push_fancy(s, x) else skip
}
```

$\leq$

```
fn push_spec(s, x) {
  atomic {
    s := (!s) :: x
  }
}
```

↑

Client $C[]$

Problem 2: Specifying Atomicity

Complex implementation | refines | simple implementation

$$\frac{push\_fancy \leq push\_spec}{\text{Behaviors}(C[push\_fancy]) \subseteq \text{Behaviors}(C[push\_spec])}$$

if b **then** $push\_fancy(s, x)$ **else skip**

}

Client $C[]$

Problem 2: Specifying Atomicity

Complex implementation refines simple implementation

$$\frac{push\_fancy \leq push\_spec \qquad \{P\}\ C[push\_spec]\ \{Q\}}{\{P\}\ C[push\_fancy]\ \{Q\}}$$

if b **then** $push\_fancy(s, x)$ **else skip**

}

Client $C[]$

Problem 2: Specifying Atomicity

Complex implementation refines simple implementation

$$\frac{push\_fancy \leq push\_spec \qquad \{P\}\ C[push\_spec]\ \{Q\}}{\{P\}\ C[push\_fancy]\ \{Q\}}$$

if b **then** $push\_fancy(s, x)$ **else skip**

}

Client $C[]$

Problem 2: Specifying Atomicity

Complex implementation — refines — simple implementation

$$
\begin{array}{l}
\textbf{fn } \mathit{push\_fancy}(s, x)\ \{ \\
\quad \textbf{let } h_n = \textbf{ref}\ (\mathsf{next} \mapsto \textbf{null}, \mathsf{value} \mapsto x)\ \textbf{in} \\
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\quad \textbf{atomic}\ \{ \\
\quad\quad s := (!s) :: x \\
\quad \} \\
\}
\end{array}
$$

↑

Client $C[]$

Problem 2: Specifying Atomicity

Complex implementation — satisfies — logically atomic specification

$$
\begin{array}{l}
\textbf{fn } \textit{push\_fancy}(s, x)\ \{ \\
\quad \textbf{let } h_n = \textbf{ref}\ (\text{next} \mapsto \textbf{null}, \text{value} \mapsto x)\ \textbf{in} \\
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\quad h_n.\text{next} := h_o; \\
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\quad \textbf{if } b \textbf{ then } \textit{push\_fancy}(s, x)\ \textbf{else skip} \\
\}
\end{array}
$$

$$\models$$

$$
\begin{array}{l}
\langle \text{Stack}(s, l) \rangle \\
\quad \textit{push\_fancy}(s, x) \\
\langle \text{Stack}(s, x :: l) \rangle
\end{array}
$$

↑

Client $C[]$

Problem 2: Specifying Atomicity

Complex implementation | satisfies | logically atomic specification

$$\dfrac{\langle \mathsf{Stack}(s, l) \rangle \; \mathit{push\_fancy}(s, x) \; \langle \mathsf{Stack}(s, x :: l) \rangle \quad \cdots \vdash \{P\} \; C[\mathit{push\_fancy}] \; \{Q\}}{\{P\} \; C[\mathit{push\_fancy}] \; \{Q\}}$$

}

Client $C[]$

Iris Contributions

1. Encoding protocols with invariants and PCMs

Iris Contributions

1. Encoding protocols with invariants and PCMs
2. Supporting logically atomic specifications
    - Defined as derived notion

Iris Contributions

1. Encoding protocols with invariants and PCMs

Do a lot with little:
Derive rules that were built-in for previous logics

Protocols = Monoids + Invariants

A simple example

$$
\begin{array}{l}
\textbf{fn } \textit{inc2}(x)\ \{ \\
\quad \textbf{do}\ \{ \\
\qquad v = !x; \\
\qquad b = \textbf{cas}(x, v, v+2); \\
\quad \}\ \textbf{while}\ (\textbf{not}\ b); \\
\}
\end{array}
$$

A simple example

fn *inc2*(x) {
  do {
    $v = !x$;
    $b = \textbf{cas}(x, v, v + 2)$;
  } **while** (**not** b);
}

Set x from v to $v + 2$

A simple example

fn *inc2*(x) {
    do {
        $v = !x$;
        $b =$ **cas**$(x, v, v + 2)$;
    } **while** (**not** b);
}

Repeat until **cas** succeeds

Invariants

"x points to an even number"

$$
\begin{array}{l}
\textbf{fn } \mathit{inc2}(x)\ \{\\
\quad \textbf{do}\ \{\\
\qquad v = !x;\\
\qquad b = \textbf{cas}(x, v, v+2);\\
\quad \}\ \textbf{while}\ (\textbf{not}\ b);\\
\}
\end{array}
$$

Invariants

"x points to an even number" $R \triangleq \exists i.\ x \mapsto i * \mathrm{ev}(i)$

$$
\begin{array}{l}
\textbf{fn}\ \mathit{inc2}(x)\ \{ \\
\quad \textbf{do}\ \{ \\
\qquad v = !x; \\
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\quad \}\ \textbf{while}\ (\textbf{not}\ b); \\
\}
\end{array}
$$

Invariants

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$\quad \textbf{do}\ \{$
$\quad\quad v = !x;$
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$\quad \}\ \textbf{while}\ (\textbf{not}\ b);$
$\}$

Before **cas**:
Obtain invariant

Invariants

"x points to an even number" $R \triangleq \exists i.\; x \mapsto i * \mathrm{ev}(i)$

fn *inc2*(x) {
  do {
    $v = !x;$
    $b =$ **cas**$(x, v, v + 2);$
  } **while** (**not** b);
}

Before **cas**:
Obtain invariant

After **cas**:
Re-establish invariant

Invariants

"x points to an even number" $R \triangleq \exists i.\ x \mapsto i * \mathsf{ev}(i)$

```
fn inc2(x) {
  do {
    v = !x;
    b = cas(x, v, v + 2);
  } while (not b);
}
```

$$\frac{\{R * P\}\ e\ \{R * Q\} \qquad e \text{ atomic}}{\boxed{R} \vdash \{P\}\ e\ \{Q\}}$$

cf. CSL [O'H07]

Invariants

"x points to an even number" $R \triangleq \exists i.\ x \mapsto i * \mathrm{ev}(i)$

$\mathbf{fn}\ \mathit{inc2}(x)\ \{$
$\quad \mathbf{do}\ \{$
$\quad\quad v = !x;$
$\quad\quad b = \mathbf{cas}(x, v, v + 2);$
$\quad \}\ \mathbf{while}\ (\mathbf{not}\ b);$
$\}$

$$\frac{\{R * P\}\ e\ \{R * Q\} \qquad e \text{ atomic}}{\boxed{R} \vdash \{P\}\ e\ \{Q\}}$$

Invariant

cf. CSL [O'H07]

Invariants

"x points to an even number" $R \triangleq \exists i.\ x \mapsto i * \mathrm{ev}(i)$

fn *inc2*(x) {
  do {
    $v = !x;$
    $b = \mathbf{cas}(x, v, v + 2);$
  } **while** (**not** b);
}

$$\frac{\{R * P\}\ e\ \{R * Q\} \quad e \text{ atomic}}{\boxed{R} \vdash \{P\}\ e\ \{Q\}}$$

Resources carried in

cf. CSL [O'H07]

Invariants

"x points to an even number" $R \triangleq \exists i.\ x \mapsto i * \mathrm{ev}(i)$

$$
\begin{array}{l}
\textbf{fn } \mathit{inc2}(x)\ \{ \\
\quad \textbf{do}\ \{ \\
\qquad v = !x; \\
\qquad b = \textbf{cas}(x, v, v + 2); \\
\quad \}\ \textbf{while}\ (\textbf{not}\ b); \\
\}
\end{array}
$$

$$\frac{\{R * P\}\ e\ \{R * Q\} \qquad e \text{ atomic}}{\boxed{R} \vdash \{P\}\ e\ \{Q\}}$$

cf. CSL [O'H07]

Resources carried out

Invariants

"x points to an even number" $R \triangleq \exists i.\ x \mapsto i * \mathrm{ev}(i)$

```
fn inc2(x) {
  do {
    v = !x;
    b = cas(x, v, v + 2);
  } while (not b);
}
```

$$\frac{\{R * P\}\ e\ \{R * Q\} \qquad e \text{ atomic}}{\boxed{R} \vdash \{P\}\ e\ \{Q\}}$$

Avoid interference

cf. CSL [O'H07]

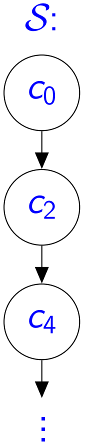
Invariants are not enough

"x points to a *monotonically increasing* even number"

$$
\begin{array}{l}
\textbf{fn}\ \mathit{inc2}(x)\ \{ \\
\quad \textbf{do}\ \{ \\
\qquad v = !x; \\
\qquad b = \textbf{cas}(x, v, v+2); \\
\quad \}\ \textbf{while}\ (\textbf{not}\ b); \\
\}
\end{array}
$$

STS Example

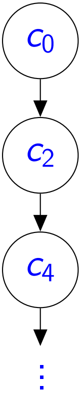
fn *inc2*(x) $\{$
    do $\{$
        $v = !x;$
        $b =$ **cas**$(x, v, v + 2);$
    $\}$ **while** (**not** b);
$\}$



STS Example

$\varphi(c_i) \triangleq x \mapsto i$

$\mathcal{S}$:




```
fn inc2(x) {
    do {
        v = !x;
        b = cas(x, v, v + 2);
    } while (not b);
}
```

STS Example

$\{\geq c_i\}$

fn *inc2*(x) {

  do {

    $v = !x;$

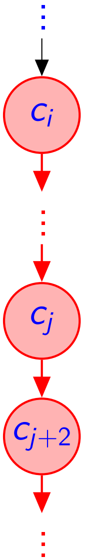
    $b =$ **cas**$(x, v, v+2);$

  } **while** (**not** b);

}

$\{\geq c_{i+2}\}$

$\varphi(c_i) \triangleq x \mapsto i$



STS Example

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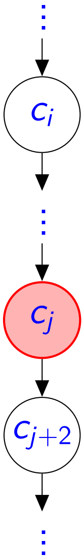
Current state: c_j



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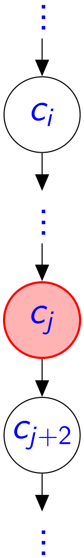
Current state: c_j,
so $x \mapsto j$



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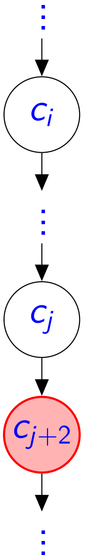
$\{\boxed{\geq c_{i+2}}\}$

Update state to c_{j+2}



STS Example

$\{\boxed{\geq c_i}\}$

$\varphi(c_i) \triangleq x \mapsto i$

```
fn inc2(x) {
    do {
        v = !x;
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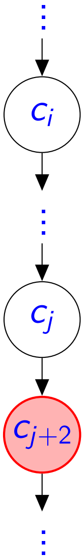
    } while (not b);
}
```

$\{\boxed{\geq c_{i+2}}\}$

Update state to c_{j+2}, show: $x \mapsto j + 2$



STS Example

$\{\geq c_i\}$

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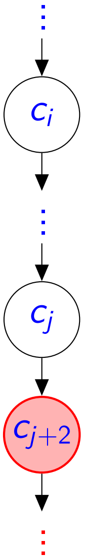
```

$\{\geq c_{i+2}\}$

$\varphi(c_i) \triangleq x \mapsto i$

Obtain $\geq c_{j+2}$



STS Example

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fn *inc2*(x) {
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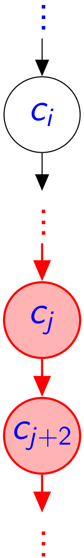
Obtain $\boxed{\geq c_{j+2}}$

$\{\boxed{\geq c_{i+2}}\}$



STS Example

$$\frac{\forall c.\ \{\hat{c} \rightarrow^* c * \varphi(c) * P\}\ e\ \{v.\ \exists c'.\ c \rightarrow^* c' * \varphi(c') * Q\}}{\mathrm{STS}(\mathcal{S}, \varphi) \vdash \{\boxed{\geq \hat{c}} * P\}\ e\ \{v.\ \exists c'.\ \boxed{\geq c'} * Q\}}$$

$\{\boxed{\geq c_i}\}$

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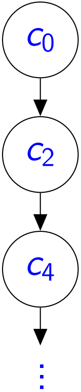

Complex rules built-in as primitives

CaReSL: $$\frac{\mathcal{C} \vdash \forall b \sqsupseteq^{\text{rely}}_{\pi} b_0.\ (\!|\pi[\![b]\!] * P|\!)\ i \Rrightarrow_1 a\ (\!|x.\ \exists b' \sqsupseteq^{\text{guar}}_{\pi} b.\ \pi[\![b']\!] * Q|\!)}{\mathcal{C} \vdash \left\{\boxed{b_0}^{n}_{\pi} * \triangleright P\right\}\ i \Rrightarrow a\ \left\{x.\ \exists b'.\ \boxed{b'}^{n}_{\pi} * Q\right\}}\ \textsc{UpdIsl}$$

All you need are two simple primitives:

- Invariants
- Partial commutative monoids

TaDA: $$\frac{a \notin \mathcal{A} \quad \forall x \in X.\ (x, f(x)) \in \mathcal{T}_{\mathbf{t}}(\mathrm{G})^* \qquad \lambda; \mathcal{A} \vdash \mathbb{V}x \in X.\ \langle p_p \mid I(\mathbf{t}^{\lambda}_a(x)) * p(x) * [\mathrm{G}]_a \rangle\ \mathbb{C}\ \exists\!\!\exists y \in Y.\ \langle q_p(x,y) \mid I(\mathbf{t}^{\lambda}_a(f(x))) * q(x,y) \rangle}{\lambda + 1; \mathcal{A} \vdash \mathbb{V}x \in X.\ \langle p_p \mid \mathbf{t}^{\lambda}_a(x) * p(x) * [\mathrm{G}]_a \rangle\ \mathbb{C}\ \exists\!\!\exists y \in Y.\ \langle q_p(x,y) \mid \mathbf{t}^{\lambda}_a(f(x)) * q(x,y) \rangle}\ \text{Use atomic rule}$$

Logical ("ghost") resources

Partial commutative monoid (PCM)

- Set M (carrier)
- An operation $\cdot$ on M (associative, commutative)
- A unit ϵ ("empty")
- A zero $\bot$ ("bottom", "undefined")

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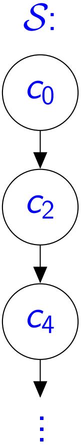
Resource $a \in M$: Logical assertion $\boxed{a}$ ("own a")

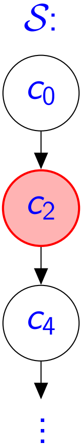
$$\frac{a \cdot b = c}{\boxed{a} * \boxed{b} \Leftrightarrow \boxed{c}} \qquad \boxed{\bot} \Rightarrow \text{False}$$

STS monoid: Intuition



STS monoid: Intuition

Curr c_2



STS monoid: Intuition

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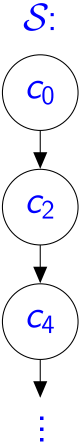
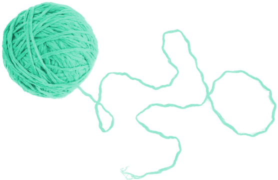


Poss $\{c_j \in \mathcal{S} \mid j \geq 2\}$

Poss $\mathcal{S}$



STS monoid: Intuition

Curr c_2

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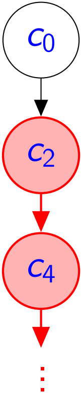

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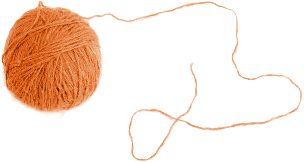

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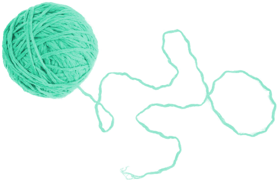

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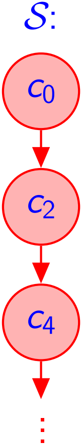

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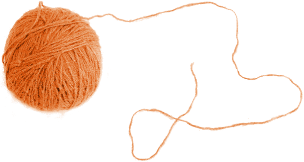
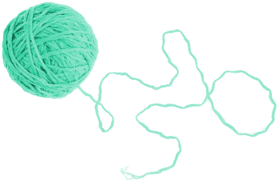
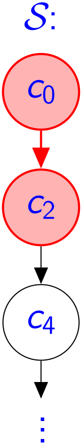




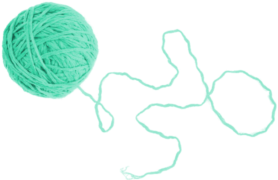
STS monoid: Intuition

Curr c_2



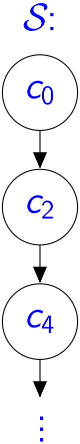
Poss $\{c_j \in \mathcal{S} \mid j \geq 2\}$

Poss $\{c_0, c_2\}$



STS monoid: Intuition

Curr c_2



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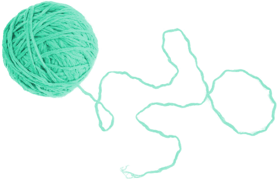
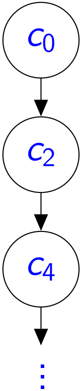


STS monoid: Formal definition

$$M \triangleq \{ \quad \} \cup \left\{ \quad \right\}$$

$\mathcal{S}$:



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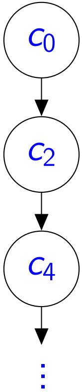

STS monoid: Formal definition

$$M \triangleq \{\mathsf{Curr}\ c \mid c \in \mathcal{S}\} \cup \left\{ \mathsf{Poss}\ B \;\middle|\; \begin{array}{l} B \subseteq \mathcal{S} \wedge B \neq \emptyset \wedge \\ \\ \end{array} \right\}$$

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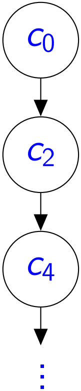

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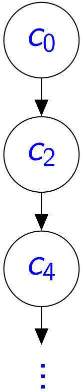

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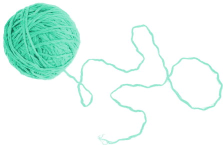
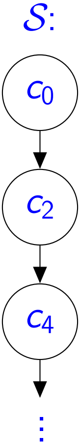
 $\parallel$  $\triangleq$ 

$$\text{Poss } B_1 \cdot \text{Poss } B_2 \triangleq \text{Poss } (B_1 \cap B_2)$$



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 $\|$  $\triangleq$ 

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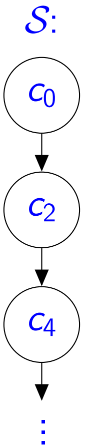
$$\text{Poss } B \cdot \text{Curr } c \triangleq \text{Curr } c \quad \text{if } c \in B$$



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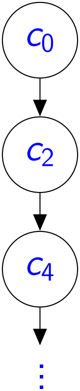
$$\mathrm{Curr}\ c_1 \cdot \mathrm{Curr}\ c_2 \triangleq \bot$$

$\mathcal{S}$:



STS invariant

STS invariant

Interaction of monoids and invariants

- Monoids serve to *express* protocols.
- Invariants serve to *enforce* protocols on shared state.

STS invariant

$$\text{Invariant } R \triangleq \exists c.\ \boxed{\mathsf{Curr}\ c} * \varphi(c)$$

Current state of STS

Interaction of monoids and invariants

- Monoids serve to *express* protocols.
- Invariants serve to *enforce* protocols on shared state.

STS invariant

Invariant $R \triangleq \exists c.\ \boxed{\text{Curr } c} * \varphi(c)$

STS interpretation

Interaction of monoids and invariants

- Monoids serve to *express* protocols.
- Invariants serve to *enforce* protocols on shared state.

Show $\{\boxed{\geq c_i}\}\ inc2(x)\ \{\boxed{\geq c_{i+2}}\}$

$$\frac{\forall c.\ \{\hat{c} \rightarrow^* c * \varphi(c) * P\}\ e\ \{v.\ \exists c'.\ c \rightarrow^* c' * \varphi(c') * Q\}}{\mathrm{STS}(\mathcal{S}, \varphi) \vdash \left\{\boxed{\geq \hat{c}} * P\right\}\ e\ \left\{v.\ \exists c'.\ \boxed{\geq c'} * Q\right\}}$$

Show $\{\boxed{\geq c_i}\}\ \mathit{inc2}(x)\ \{\boxed{\geq c_{i+2}}\}$

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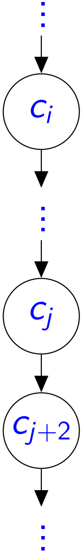
STS reasoning

$R \triangleq \exists c_i.\ \boxed{\text{Curr } c_i} * x \mapsto i$



$\{\boxed{\geq c_i}\}$

fn *inc2*(x) {

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  } **while** (**not** b);

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$\{\boxed{\geq c_{i+2}}\}$

STS reasoning

$B_i \triangleq \{c_j \in \mathcal{S} \mid j \geq i\}$ $\quad$ $R \triangleq \exists c_i.\ \boxed{\text{Curr } c_i} * x \mapsto i$

$\{\boxed{\text{Poss } B_i}\}$

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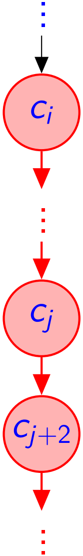

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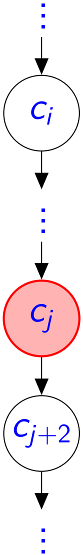
```

$\{\boxed{\text{Poss } B_{i+2}}\}$

Obtain R:
$\boxed{\text{Poss } B_i} * \boxed{\text{Curr } c_j} * x \mapsto j$



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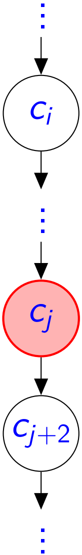
Obtain R:

$\boxed{\text{Poss } B_i} * \boxed{\text{Curr } c_j} * x \mapsto j$

Remember

$\text{Poss } B_i \cdot \text{Curr } c_j \triangleq \text{Curr } c_j$

if $c_j \in B_i$



STS reasoning

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    v = !x;
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}
```

$\{\boxed{\text{Poss } B_{i+2}}\}$

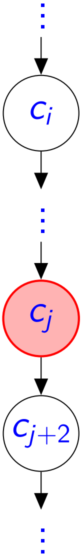
So we have:

$c_j \in B_i * \boxed{\text{Curr } c_j} * x \mapsto j$

Remember

$\text{Poss } B_i \cdot \text{Curr } c_j \triangleq \text{Curr } c_j$

if $c_j \in B_i$



STS reasoning

$B_i \triangleq \{c_j \in \mathcal{S} \mid j \geq i\}$ $R \triangleq \exists c_i.\ \boxed{\text{Curr } c_i} * x \mapsto i$

$\{\boxed{\text{Poss } B_i}\}$

fn *inc2*(x) {
  do {
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    $b =$ **cas**$(x, v, v + 2)$;
  } **while** (**not** b);
}

$\{\boxed{\text{Poss } B_{i+2}}\}$

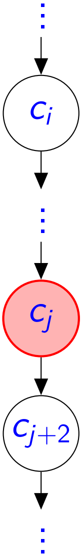
So we have:

$j \geq i * \boxed{\text{Curr } c_j} * x \mapsto j$

Remember

$\text{Poss } B_i \cdot \text{Curr } c_j \triangleq \text{Curr } c_j$

if $c_j \in B_i$



STS reasoning

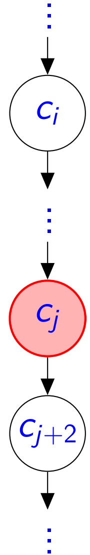
$B_i \triangleq \{c_j \in \mathcal{S} \mid j \geq i\}$ $\quad$ $R \triangleq \exists c_i.\ \boxed{\text{Curr } c_i} * x \mapsto i$

$\{\boxed{\text{Poss } B_i}\}$

fn *inc2*(x) {
    do {
        $v = !x;$
        $b =$ **cas**$(x, v, v + 2);$
    } **while** (**not** b);
}

$\{\boxed{\text{Poss } B_{i+2}}\}$

We have:
$\boxed{\text{Curr } c_j} * x \mapsto j + 2,$
We want: R



STS reasoning

$B_i \triangleq \{c_j \in \mathcal{S} \mid j \geq i\}$ $\quad$ $R \triangleq \exists c_i.\ \boxed{\text{Curr } c_i} * x \mapsto i$

$\{\boxed{\text{Poss } B_i}\}$

fn *inc2*(x) {

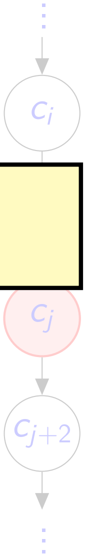
We need a way to update $\boxed{\text{Curr } c_j}$ to $\boxed{\text{Curr } c_{j+2}}$

$b =$ **cas**$(x, v, v+2)$;

} **while** (**not** b);

}

$\{\boxed{\text{Poss } B_{i+2}}\}$



STS reasoning

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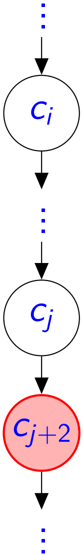
$\{\boxed{\text{Poss } B_i}\}$

fn $inc2(x)$ {
  do {
    $v = !x;$
    $b = \textbf{cas}(x, v, v+2);$
  } **while** (**not** b);
}

$\{\boxed{\text{Poss } B_{i+2}}\}$

We have R:
$\boxed{\text{Curr } c_{j+2}} * x \mapsto j + 2$



STS reasoning

$B_i \triangleq \{c_j \in \mathcal{S} \mid j \geq i\}$ $\quad$ $R \triangleq \exists c_i.\ \boxed{\text{Curr } c_i} * x \mapsto i$

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fn *inc2*(x) {
 do {
  $v = {!}x$;
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 } **while** (**not** b);
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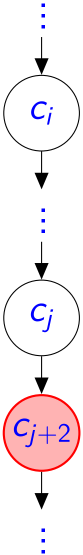
$\{\boxed{\text{Poss } B_{i+2}}\}$

We have R:
$\boxed{\text{Curr } c_{j+2}} * x \mapsto j + 2$,
We want: $\boxed{\text{Poss } B_{i+2}}$



STS reasoning

$B_i \triangleq \{c_j \in \mathcal{S} \mid j \geq i\}$ $\quad R \triangleq \exists c_i.\ \boxed{\text{Curr } c_i} * x \mapsto i$

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$\{\boxed{\text{Poss } B_{i+2}}\}$

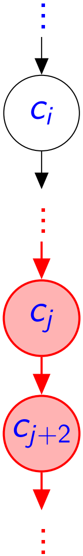
We have R:
$\boxed{\text{Curr } c_{j+2}} * x \mapsto j + 2,$
We want: $\boxed{\text{Poss } B_{i+2}}$

Remember

$\text{Poss } B_{i+2} \cdot \text{Curr } c_{j+2}$

$\triangleq \text{Curr } c_{j+2}$

if $c_{j+2} \in B_{i+2}$



Frame-preserving ghost update

$$\boxed{\text{Curr } c_j} \Rrightarrow \boxed{\text{Curr } c_{j+2}}$$

Frame-preserving ghost update

$$\boxed{a} \Rightarrow \boxed{b}$$

Frame-preserving ghost update

$$\boxed{a} \Rrightarrow \boxed{b}$$

	Us	vs.	the world
We always have	a	$\#$	a_f (for some *frame*)

where $a \# a_f \triangleq a \cdot a_f \neq \bot$

Frame-preserving ghost update

$$\boxed{a} \Rrightarrow \boxed{b}$$

	Us	vs.	the world
We always have	a	$\#$	a_f (for some *frame*)
So if we can show	b	$\#$	a_f

where $a \# a_f \triangleq a \cdot a_f \neq \bot$

Frame-preserving ghost update

$$\boxed{a} \Rrightarrow \boxed{b}$$

	Us	vs.	the world
We always have	a	$\#$	a_f (for some *frame*)
So if we can show	b	$\#$	a_f
We obtain	$\boxed{a} \Rrightarrow \boxed{b}$		

where $a \# a_f \triangleq a \cdot a_f \neq \bot$

Frame-preserving ghost update

$$\frac{\forall a_f.\ a \mathrel{\#} a_f \Rightarrow b \mathrel{\#} a_f}{\boxed{a} \Rrightarrow \boxed{b}}$$

cf. Views [DY+13]

	Us	vs.	the world
We always have	a	$\#$	a_f (for some *frame*)
So if we can show	b	$\#$	a_f
We obtain	$\boxed{a} \Rrightarrow \boxed{b}$		

where $a \mathrel{\#} a_f \triangleq a \cdot a_f \neq \bot$

Frame-preserving ghost update

$$\boxed{\text{Curr } c_j} \Rightarrow \boxed{\text{Curr } c_{j+2}}$$

Frame-preserving ghost update

$$\frac{c \to^* c'}{\boxed{\mathsf{Curr}\ c} \Rrightarrow \boxed{\mathsf{Curr}\ c'}}$$

Frame-preserving ghost update

$$\frac{c \to^* c'}{\boxed{\mathrm{Curr}\ c} \Rrightarrow \boxed{\mathrm{Curr}\ c'}}$$

	Us	vs.	the world
We have	$\mathrm{Curr}\ c$	$\#$	a_f

Frame-preserving ghost update

$$\frac{c \to^* c'}{\boxed{\text{Curr } c} \Rrightarrow \boxed{\text{Curr } c'}}$$

	Us	vs.	the world
We have	$\text{Curr } c$	$\#$	?

Remember

$$\text{Curr } c_1 \cdot \text{Curr } c_2 \triangleq \bot$$

$\parallel$

Frame-preserving ghost update

$$\frac{c \to^* c'}{\boxed{\mathsf{Curr}\ c} \Rrightarrow \boxed{\mathsf{Curr}\ c'}}$$

	Us	vs.	the world
We have	$\mathsf{Curr}\ c$	$\#$	$\mathsf{Poss}\ B$

Remember

$$\mathsf{Poss}\ B \cdot \mathsf{Curr}\ c \triangleq \mathsf{Curr}\ c \quad \text{if } c \in B$$

 $\parallel$  $\triangleq$ 

Frame-preserving ghost update

$$\frac{c \to^* c'}{\boxed{\mathsf{Curr}\ c} \Rrightarrow \boxed{\mathsf{Curr}\ c'}}$$

Us	vs.	the world
We have $\mathsf{Curr}\ c$	$\#$	$\mathsf{Poss}\ B$, $c \in B$

Remember

$$\mathsf{Poss}\ B \cdot \mathsf{Curr}\ c \triangleq \mathsf{Curr}\ c \quad \text{if } c \in B$$

 $\|$  $\triangleq$ 

Frame-preserving ghost update

$$\frac{c \to^* c'}{\boxed{\text{Curr } c} \Rrightarrow \boxed{\text{Curr } c'}}$$

	Us	vs.	the world
We have	$\text{Curr } c$	$\#$	$\text{Poss } B$, $c \in B$
Show	$\text{Curr } c'$	$\#$	$\text{Poss } B$: $c' \in B$

Remember

$$\text{Poss } B \cdot \text{Curr } c \triangleq \text{Curr } c \quad \text{if } c \in B$$

 $\|$  $\triangleq$ 

Frame-preserving ghost update

$$\frac{c \to^* c'}{\boxed{\text{Curr } c} \Rrightarrow \boxed{\text{Curr } c'}}$$

	Us	vs.	the world
We have	$\text{Curr } c$	$\#$	$\text{Poss } B$, $c \in B$
Show	$\text{Curr } c'$	$\#$	$\text{Poss } B$: $c' \in B$

Remember

$$M \triangleq \{\text{Curr } c \mid c \in \mathcal{S}\} \cup \left\{ \text{Poss } B \;\middle|\; \begin{array}{l} B \subseteq \mathcal{S} \wedge B \neq \emptyset \wedge \\ B \text{ closed under } \to \end{array} \right\}$$

Frame-preserving ghost update

$$\frac{c \to^* c'}{\boxed{\text{Curr } c} \Rrightarrow \boxed{\text{Curr } c'}}$$

	Us	vs.	the world
We have	Curr c	$\#$	Poss B, $c \in B$
Show	Curr c'	$\#$	Poss B: $c' \in B$

$$\boxed{\text{Curr } c} \Rrightarrow \boxed{\text{Curr } c'}$$

Summary: Rules for PCMs and invariants

$$\frac{\{R * P\}\ e\ \{R * Q\} \qquad e \text{ atomic}}{\boxed{R} \vdash \{P\}\ e\ \{Q\}}$$

$$\frac{\forall a_f.\ a \# a_f \Rightarrow b \# a_f}{\boxed{a} \Rrightarrow \boxed{b}}$$

$$\frac{a \cdot b = c}{\boxed{a} * \boxed{b} \Leftrightarrow \boxed{c}}$$

$$\boxed{\bot} \Rightarrow \text{False}$$

Summary: Rules for PCMs and invariants

We can derive the STS update rule

$$\frac{\forall c.\ \{\hat{c} \rightarrow^* c * \varphi(c) * P\}\ e\ \{v.\ \exists c'.\ c \rightarrow^* c' * \varphi(c') * Q\}}{\mathsf{STS}(\mathcal{S}, \varphi) \vdash \left\{\boxed{\geq \hat{c}} * P\right\}\ e\ \left\{v.\ \exists c'.\ \boxed{\geq c'} * Q\right\}}$$

using just monoids and invariants.

What else is in the paper?

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- Contribution 2: Logically atomic specifications
  - à la TaDA by da Rocha Pinto, Dinsdale-Young, and Gardner [dDYG14]
  - Defined as derived form
  - Can handle helping

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  - Defined as derived form
  - Can handle helping
- More constructions based on monoids and invariants
- Coq mechanization

Case study: Stack of abstractions

Elimination stack

Shared memory

Message-passing machine

Case study: Stack of abstractions

Elimination stack

You can do a lot with very little.

Message-passing machine

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Refinement and Hoare specs

$$e_1 \triangleq \textbf{let } x := \textbf{ref } 7 \textbf{ in } 42$$
$$e_2 \triangleq 42$$

Clearly, $e_2 \leq e_1$ and

$$\{\mathsf{True}\}\ e_1\ \{\exists x.\ x \mapsto 7\}$$

But this does not hold:

$$\{\mathsf{True}\}\ e_2\ \{\exists x.\ x \mapsto 7\}$$